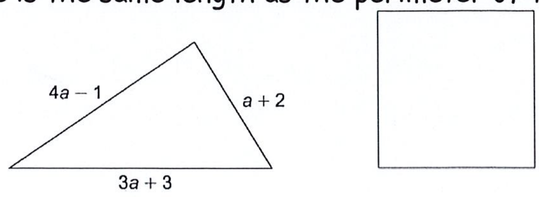
ALGEBRA

Collecting Like Terms

The perimeter of the triangle is the same length as the perimeter of the square.



Find an expression for the length of one side of the square in terms of a.
Give your answer in its simplest form.

$4a-1 + a+2 + 3a+3 = 8a+4$

$\frac{8a+4}{4} = 2a+1$

.........$2a+1$.........

Substitution

Complete the table for $y = x^3 - 6x - 5$.

x	0	1	2	3	4
y	-5	-10	-9	4	35

$0^3 - 6 \times 0 - 5 = -5$

$4^3 - 6 \times 4 - 5 = 64 - 24 - 5 = 35$

Laws of Indices

a) Simplify

$\left(\frac{x^4y}{x^2y^2}\right)^3 = \left(\frac{x^2}{y}\right)^3$

.........$\frac{x^6}{y^3}$.........

b) Show that $\sqrt[3]{a^4} \times \frac{1}{a}$ can be expressed as $a^{\frac{1}{3}}$

$a^{\frac{4}{3}} \times a^{-1} = a^{\frac{4}{3}-1} = a^{\frac{4}{3}-\frac{3}{3}} = a^{\frac{1}{3}}$

c) Simplify fully:

$\frac{3a^8 \times 2a^5}{a^2} = \frac{6a^{13}}{a^2}$

.........$6a^{11}$.........

Expanding and Simplifying

Expand and simplify:

a) $2(3x + 5) + 3(x + 1)$

$6x + 10 + 3x + 3$

$= 6x + 3x + 10 + 3$

$9x + 13$

b) $3x(3x - 5) - 2x(x - 8)$

$9x^2 - 15x - (2x^2 - 16x)$

$= 9x^2 - 15x - 2x^2 + 16x$

$= 9x^2 - 2x^2 + 16x - 15x$

$7x^2 + x$

Factorising Expressions

Factorise fully:

a) $6x - x^2$

$6x(1 - x)$

b) $10xy - 25x$

$5x(2y - 5)$

c) $18abc + 24bc - 12b$

$6b(3ac + 4c - 2)$

Expanding Double Brackets

Expand and simplify:

a) $(x + 7)(x + 2)$

	x	$+7$
x	x^2	$+7x$
$+2$	$+2x$	$+14$

$x^2 + 9x + 14$

b) $(2x + 1)(x - 5)$

	$2x$	$+1$
x	$2x^2$	$+x$
-5	$-10x$	-5

$2x^2 - 9x - 5$

c) $(3x - y)(2x + 5y)$

	$3x$	$-y$
$2x$	$6x^2$	$-2xy$
$+5y$	$+15xy$	$-5y^2$

$6x^2 + 13xy - 5y^2$

Factorising Quadratics

Factorise fully:

a) $x^2 + 5x - 24$

$(x+8)(x-3)$

b) $5x^2 + 7x + 2$

$(5x+2)(x+1)$

c) $x^2 - 49$

$(x+7)(x-7)$

Products of Three Binomials

Expand and simplify.

$$(2x - 1)(x + 5)(3x - 2)$$

	$2x$	-1
x	$2x^2$	$-x$
$+5$	$+10x$	-5

$= 2x^2 + 9x - 5$

	$2x^2$	$+9x$	-5
$3x$	$6x^3$	$+27x^2$	$-15x$
-2	$-4x^2$	$-18x$	$+10$

$6x^3 + 23x^2 - 33x + 10$

Rearranging Formulae

a) Make x the subject of the formula $y = \sqrt{4x - 3}$

$y^2 = 4x - 3$

$+3 \quad +3$

$y^2 + 3 = 4x$

$\div 4 \quad \div 4$

$\frac{y^2+3}{4} = x$

$x = \frac{y^2+3}{4}$

b) Make a the subject of the formula $s = ut + \frac{1}{2}at^2$

$-ut \quad -ut$

$s - ut = \frac{1}{2}at^2$

$\times 2 \quad \times 2$

$2s - 2ut = at^2$

$\div t^2 \quad \div t^2$

$\frac{2s - 2ut}{t^2} = a$

$a = \frac{2s - 2ut}{t^2}$

c) Show that $k = \frac{4+3j}{5-j}$ can be rearranged to give $j = \frac{5k-4}{3+k}$

$k(5 - j) = 4 + 3j$

$5k - jk = 4 + 3j$

$+jk \quad +jk$

$5k = 4 + 3j + jk$

$-4 \quad -4$

$5k - 4 = 3j + jk$

$5k - 4 = j(3 + k)$

$\frac{5k-4}{3+k} = j$

Solving Equations

Solve:

a) $3x - 4 = \frac{x}{2}$

$\times 2 \quad \times 2$

$6x - 8 = x$

$-x \quad -x$

$5x - 8 = 0$

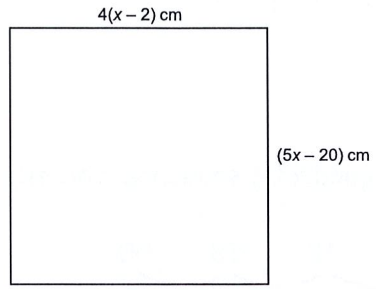
$+8 \quad +8$

$5x = 8$

$\div 5 \quad \div 5$

$x = \frac{8}{5}$

b) This is a square:



Not to scale

Work out the length of the side of the square.

$4(x-2) = 5x - 20$

$4x - 8 = 5x - 20$

$-4x \quad -4x$

$-8 = x - 20$

$+20 \quad +20$

$12 = x$

$x = 12$

Linear Sequences

Here are the first four terms of a sequence.

$$\underbrace{-1 \quad 4}_{5} \quad \underbrace{\ \ 9}_{5} \quad \underbrace{\ \ 14}_{5}$$

Write an expression for the nth term of this sequence.

5n 5 10 15 20

5n − 6

Quadratic Sequences

a) This expression can be used to generate a sequence of numbers.

$$n^2 - n + 11$$

Work out the first three terms of this sequence.

$n=1$ $\quad 1^2 - 1 + 11 = 11$

$n=2$ $\quad 2^2 - 2 + 11 = 13$

$n=3$ $\quad 3^2 - 3 + 11 = 17$

........11, 13, 17........

Show that this expression does not only generate prime numbers.

$n=11$ $\quad 11^2 - 11 + 11 = 121$, not prime.

b) Here are the first four terms of a quadratic sequence, the nth term of this quadratic sequence is $an^2 + bn + c$.

2 12 28 50

10 16 22

6 6

Find the values of a, b and c.

$3n^2$	3	12	27	48	
	2	12	28	50	
+	-1	0	+1	+2	$n-2$

$3n^2 + n - 2$

a = ...3........

b = ...1........

c = ...-2........

Other Sequences

a) Here is a sequence: $2, 2\sqrt{7}, 14, 14\sqrt{7}$

Work out the next term.

$14\sqrt{7} \times \sqrt{7} = 14 \times 7 = 98$

........98........

a) Here is a sequence: $\underbrace{5, 5\sqrt{3}}_{\times\sqrt{3}}, \underbrace{15}_{\times\sqrt{3}}, \underbrace{15\sqrt{3}}_{\times\sqrt{3}}$

Write the nth term.

$\sqrt{3}^n = \sqrt{3}, 3, 3\sqrt{3}, 9, \ldots$

$\sqrt{3}^{n-1} = 1, \sqrt{3}, 3, 3\sqrt{3}$

........$5 \times \sqrt{3}^{n-1}$........

Linear Graphs

P has coordinates (0, -1) and Q has coordinates (4, 1).
Find the equation of the line PQ.

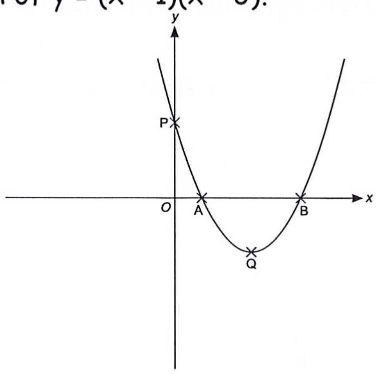
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - -1}{4 - 0} = \frac{2}{4} = \frac{1}{2}$

$c = -1 \quad (0, -1)$

$y = \frac{1}{2}x - 1$

Non-Linear Graphs

a) This is a sketch of the graph of y = (x – 1)(x – 3).



Find the coordinates of the points A, B, P and Q.

$(x-1)(x-3) = 0$

$x = 1, \quad x = 3$

A is at (1, 0) B is at (3, 0)

	x	-1
x	x^2	$-x$
-3	$-3x$	$+3$

$= x^2 - 4x + 3$

$x = 0, \quad y = 0^2 - 4 \times 0 + 3 = 3$

$P = (0, 3)$

$x^2 - 4x + 3 = (x-2)^2$ —

	x	-2
x	x^2	$-2x$
-2	$-2x$	$+4$

$= x^2 - 4x + 4$

$x^2 - 4x + 3 = (x-2)^2 - 1$

$Q = (2, -1)$

A = (1, 0)

B = (3, 0)

P = (0, 3)

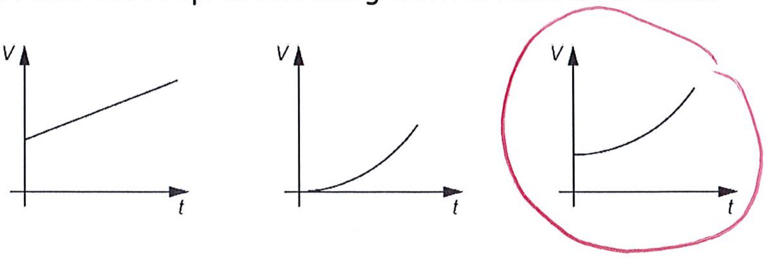
Q = (2, -1)

Reciprocal and Exponential Graphs

a) Rashid invests money into an account which pays a fixed rate of compound interest each year.
The value, £V, of his investment after t years is given by the formula

$$V = 1250 \times 1.03^t$$

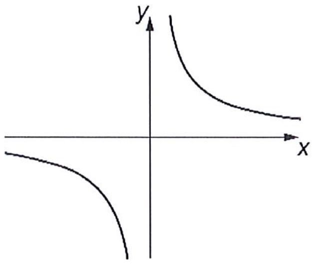
Circle the graph that best represents the growth in Rashid's account.



b) For each graph, choose its possible equation from this list:

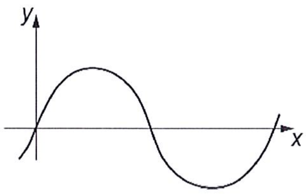
$y = \frac{1}{x}$ $\quad y = \cos x$ $\quad y = x^2$ $\quad y = 2^x$ $\quad y = \sin x$ $\quad y = x^3$ $\quad y = \frac{1}{x^2}$

i)



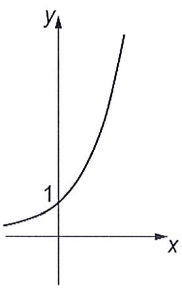
$y = \frac{1}{x}$

ii)



$y = \sin x$

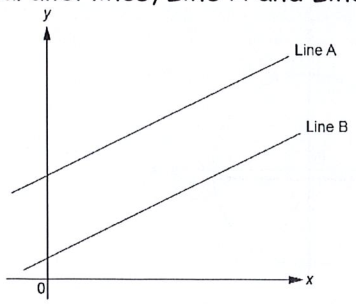
iii)



$y = 2^x$

Parallel and Perpendicular Lines

a) The graph shows two parallel lines, Line A and Line B.



Line A has equation y = 6x + 7.
Line B passes through the point (4, 26).

Find the equation of Line B.

$y = 6x$ ___

$(4, 26) \longrightarrow 26 = 6 \times 4$ ___

$26 = 24 \ \underline{+2}$

$y = 6x + 2$

b) P is the point (0, -1) and Q is the point (5, 9).
Find the equation of the line through P that is perpendicular to the line PQ.

Gradient of PQ $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - {}^-1}{5 - 0} = \frac{10}{5} = 2$

Perpendicular to PQ $= \frac{-1}{2}$

$y = \frac{-1}{2}x$ ___

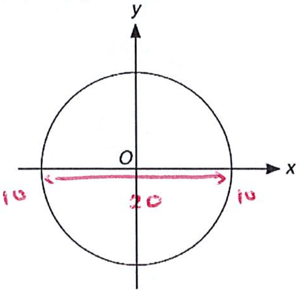
$(0, -1) \longrightarrow -1 = \frac{-1}{2} \times 0$ ___

$-1 = 0 \ \underline{-1}$

$y = \frac{-1}{2}x - 1$

Equations and Tangents of Circles

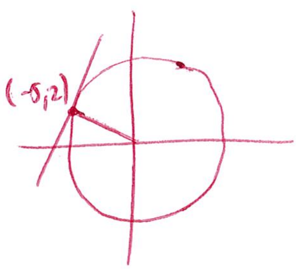
a) The diagram shows a circle, centre O.



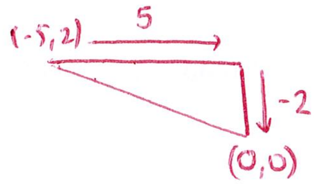
The circumference of the circle is 20π cm.
Find the equation of the circle.

$x^2 + y^2 = 100$

b) The point (-5, 2) lies on the circumference of a circle, centre (0, 0).
Work out the equation of the tangent to circle at (-5, 2).



Gradient of radius = $\cancel{\frac{2}{-5}}$ $\frac{-2}{5}$



Gradient of tangent = $\frac{5}{2}$

$y = \frac{5}{2}x$ ___

$(-5,2) \rightarrow 2 = \frac{5}{2} \times -5$ ___

$2 = -12.5$ $\underline{+14.5}$

$y = \frac{5}{2}x + \frac{29}{2}$

Forming and Solving Equations

a) Alexander, Reiner and Wim each watch a different film.
Alexander's film is thirty minutes longer than Wim's film.
Reiner's film is twice as long as Wim's film.
Altogether the films last 390 minutes.

How long is each of their films?

Wim = w

Reiner = $2w$

Alexander = $w + 30$

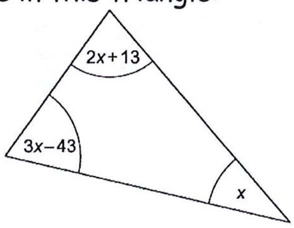
$$w + 2w + w + 30 = 390$$
$$4w + 30 = 390$$
$$4w = 360$$
$$w = 90$$

A: 120 minutes

R: 180 minutes

W: 90 minutes

b) Calculate the size of each angle in this triangle:



Not to scale

$x = 35$

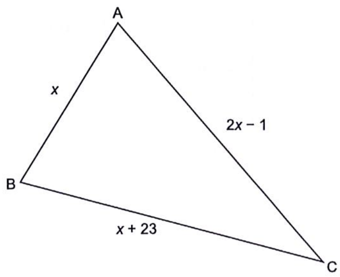
$2x + 13 = 83$

$3x - 43 = 62$

$$2x + 13 + 3x - 43 + x = 180$$
$$6x - 30 = 180$$
$$6x = 210$$
$$x = 35$$

35°, 83°, 62°

c) Triangle ABC has sides x, x + 23, and 2x − 1.



Not to scale

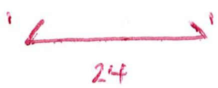
Show that there is only one value of x which makes triangle ABC isosceles.

Either $x = x + 23$ or $x = 2x - 1$ or $2x - 1 = x + 23$

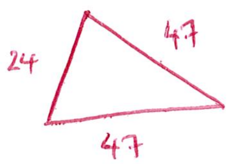
No solutions.

$x = 2x - 1$, $+1$ $+1$: $x + 1 = 2x$, $-x$ $-x$: $1 = x$

$2x - 1 = x + 23$, $-x$ $-x$: $x - 1 = 23$, $+1$ $+1$: $x = 24$



Not a triangle!



$x = 24$.

Solving Quadratic Equations

a) Solve $3x^2 = 75$

$x^2 = 25$

$x = -5$ or $x + 5$

b) Solve by factorisation $3x^2 + 11x - 20 = 0$

$(3x \quad)(x \quad)$

-20	
1	-20
-1	20
2	-10
-2	10

4 -5

-4 5 $\longrightarrow (3x - 4)(x + 5) = 0$

$3x - 4 = 0$ or $x + 5 = 0$

$3x = 4$ $\quad x = -5$

$x = \frac{4}{3}$

$x = \frac{4}{3}$ or $x = -5$.

c) $x^2 - 6x + 15 = 3x - 5$

$-3x \quad -3x$

$x^2 - 9x + 15 = -5$

$+5 \quad +5$

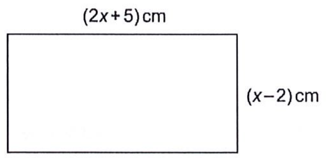
$x^2 - 9x + 20 = 0$

$(x - 4)(x - 5) = 0$

$x - 4 = 0$ or $x - 5 = 0$

$x = 4$ or $x = 5$

d) The rectangle has a length $(2x + 5)$ cm and width $(x - 2)$ cm.



Not to scale

The rectangle has an area of $35cm^2$.
Use algebra to find the value of x.

	$2x$	$+5$
x	$2x^2$	$+5x$
-2	$-4x$	-10

$2x^2 + x - 10 = 35$

$2x^2 + x - 45 = 0$

$(2x - 9)(x + 5) = 0$

$2x - 9 = 0$ or $x + 5 = 0$

$2x = 9$ $\quad x = -5$

$x = 4.5$

$x = -5$ gives a length of $x - 2 = -7$. Can't have negative lengths.

$x = 4.5$

The Quadratic Formula

a) Solve this equation.
Give each value correct to 2 decimal places.

$3x^2 + 2x - 3 = 0$

$a = 3 \quad b = 2 \quad c = -3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times -3}}{2 \times 3}$

$x = \frac{-2 \pm \sqrt{4 - {}^-36}}{6}$

$x = \frac{-2 \pm \sqrt{40}}{6}$

(+) $x = 0.72$

(−) $x = -1.39$

$x = 0.72$ or $x = -1.39$

b) Ryan is using the quadratic formula to solve an equation of the form $ax^2 + bx + c = 0$.
After substituting values into the quadratic formula, he gets

$$x = \frac{-3 \pm 3\sqrt{5}}{2}.$$

Find a possible set of values for a, b and c.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad x = \frac{-3 \pm 3\sqrt{5}}{2}$

$-b = -3 \quad b = 3$

$2a = 2 \quad a = 1$

$\sqrt{b^2 - 4ac} = 3\sqrt{5}$

$\sqrt{b^2 - 4ac} = \sqrt{9} \times \sqrt{5} = \sqrt{45}$

$b^2 - 4ac = 45$

$3^2 - 4 \times 1 \times c = 45$

$9 - 4c = 45$

$+4c \quad +4c$

$9 = 45 + 4c$

$-45 \quad -45$

$-36 = 4c$

$-9 = c$

a = 1

b = 3

c = −9

Completing the Square

a) Write $x^2 + 4x - 16$ in the form $(x + a)^2 - b$

$(x+2)^2 = x^2 + 4x + 4$

$x^2 + 4x - 16 = (x+2)^2 - 20$

$(x+2)^2 - 20$

Hence, or otherwise, find the turning point of the graph of $y = x^2 + 4x - 1$

$(-2, -20)$

b) Solve the equation $x^2 + 4x - 16 = 0$.
Give your answers in surd form as simply as possible.

$(x+2)^2 - 20 = 0$

$(x+2)^2 = 20$

$(x+2) = \pm\sqrt{20}$

$x = -2 \pm \sqrt{20}$

$x = -2 \pm 2\sqrt{5}$

$x = -2 \pm 2\sqrt{5}$

Solving Linear Inequalities

Solve:

a) $3x - 2 > 10$

$+2 \quad +2$

$3x > 12$

$\div 3 \quad \div 3$

$x > 4$

$x > 4$

b) $3x - 5 \geq 10$

$+5 \quad +5$

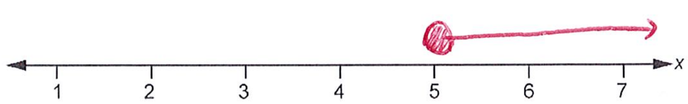
$3x \geq 15$

$\div 3 \quad \div 3$

$x \geq 5$

$x \geq 15$

Show your solution on the number line:

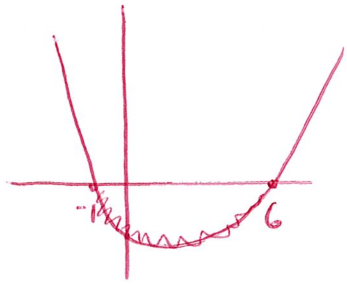


Solving Quadratic Inequalities

Solve:

a) $x^2 - 5x - 6 \leq 0$

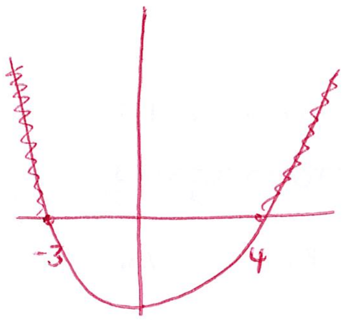
$(x-6)(x+1) \leq 0$



$-1 \leq x \leq 6$

b) $x^2 - x - 12 \geq 0$

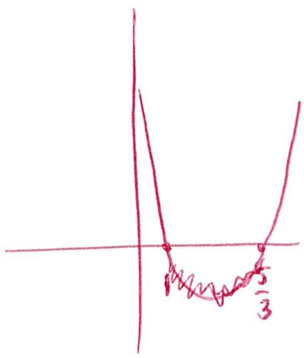
$(x-4)(x+3) \geq 0$



$x \leq -3, \quad x \geq 4.$

c) $3x^2 - 8x + 5 < 0$

$(3x-5)(x-1) < 0$



$1 < x < \frac{5}{3}$

Linear Simultaneous Equations

a) Solve

$$\begin{aligned} & 4x + 3y = 5 \\ + \quad & 2x - 3y = 1 \\ \hline & 6x = 6 \\ & x = 1 \end{aligned}$$

$x = 1 \longrightarrow 4 \times 1 + 3y = 5$

$4 + 3y = 5$
$-4 \quad\quad -4$

$3y = 1$

$y = \frac{1}{3}$

$x = 1, \; y = \frac{1}{3}$

b) Eddie and Caroline are going to the school play.
Eddie buys 6 adult tickets and 2 child tickets. He pays £39.
Caroline buys 5 adult tickets and 3 child tickets. She pays £36.50.

Work out the cost of an adult ticket and the cost of a child ticket.
(x) (y)

$6x + 2y = 39 \quad \times 3$

$5x + 3y = 36.50 \quad \times 2$

$$\begin{aligned} & 18x + 6y = 117 \\ - \quad & 10x + 6y = 73 \\ \hline & 8x = 44 \\ & x = 5.5 \end{aligned}$$

$6x = 33$

$33 + 2y = 39$

$2y = 6$

$y = 3$

Adult: £ 5.50

Child: £ 3.00

Quadratic Simultaneous Equations

Solve:

$y = x - 3$
$y = 2x^2 + 8x - 7$

$2x^2 + 8x - 7 = x - 3$
$-x \quad -x$
$2x^2 + 7x - 7 = -3$
$+3 \quad +3$
$2x^2 + 7x - 4 = 0$
$(2x - 1)(x + 4) = 0$
$2x - 1 = 0$ or $x + 4 = 0$
$2x = 1 \qquad x = -4$
$x = \frac{1}{2}$

$x = \frac{1}{2}$ or $x = -4$.

Iteration

a) A sequence is defined by the term-to-term rule $u_{n+1} = u_n^2 - 8u_n + 17$.
Given that $u_1 = 4$, find u_2 and u_3.

$u_2 = u_1^2 - 8 \times u_1 + 17 = 4^2 - 8 \times 4 + 17 = 1$
$u_3 = u_2^2 - 8 \times u_2 + 17 = 1^2 - 8 \times 1 + 17 = 10$

$u_2 =$ 1

$u_3 =$ 10

b) Show that one solution of the equation $x^3 + 2x - 5 = 0$ lies between 1 and 2.

$x = 1 \qquad 1^3 + 2 \times 1 - 5 = 1 + 2 - 5 = -2$

$x = 2 \qquad 2^3 + 2 \times 2 - 5 = 8 + 4 - 5 = 7$

0 is between -2 and 7, so x is between 1 and 2.

Functions and Inverse Functions

Two functions, f and g, are represented by these function machines:

Function f: Input → $\times 3$ → $+ 6$ → Output

Function g: Input → $+ 2$ → $\times 8$ → Output

A number is chosen.
This number is put into both function f and function g.
The output from both functions is the same.
Work out the number that was chosen.

$f: 3x + 6$

$g: 8(x+2)$

$3x + 6 = 8(x+2)$

$3x + 6 = 8x + 16$

$-3x \quad -3x$

$6 = 5x + 16$

$-16 \quad -16$

$-10 = 5x$

$\div 5 \quad \div 5$

$-2 = x$

.......-2.......

Composite Functions

Here is a function:

Function A: x → $+ 3$ → $\times 2$ → y

Here is another function:

Function B: x → $\div 5$ → $+ 4$ → y

Here is a composite function:

m → Function A → Function B → $2p + 4$

Find an expression for m in terms of p.
Give your answer in its simplest form.

$m \rightarrow m+3 \rightarrow 2m+6 \rightarrow$

$2m+6 \rightarrow \frac{2m+6}{5} \rightarrow \frac{2m+6}{5} + 4 \rightarrow$

$\frac{2m+6}{5} + 4 = 2p + 4$

$-4 \quad -4$

$\frac{2m+6}{5} = 2p$

$\times 5 \quad \times 5$

$2m + 6 = 10p$

$-6 \quad -6$

$2m = 10p - 6$

$\div 2 \quad \div 2$

$m = 5p - 3$

.......$m = 5p - 3$.......

Simplifying Algebraic Fractions

Show that $\frac{2x^2+13x+20}{2x^2+x-10}$ simplifies to $\frac{x+a}{x+b}$ where a and b are integers.

$$\frac{\cancel{(2x+5)}(x+4)}{\cancel{(2x+5)}(x-2)} = \frac{x+4}{x-2}.$$

Calculating with Algebraic Fractions

a) Show that $\frac{x+9}{x^2-1} \div \frac{4}{x+1}$ can be written in the form $\frac{a}{x-1}$ where a is an integer.

$$\frac{x+9}{\cancel{(x+1)}(x-1)} \times \frac{\cancel{(x+1)}}{4}$$

$$\frac{x+9}{4(x-1)}$$

b) Solve this equation, giving your answers correct to 1 decimal place.

$$\frac{5}{x+2} + \frac{3}{x-3} = 2$$

$$\frac{5(x-3)}{(x+2)(x-3)} + \frac{3(x+2)}{(x+2)(x-3)} = 2$$

$$\frac{5x-15 + 3x+6}{x^2-x-6} = 2$$

$$\frac{8x-9}{x^2-x-6} = 2$$

$$8x-9 = 2x^2-2x-12$$

$$+9 \qquad +9$$

$$8x = 2x^2-2x-3$$

$$-8x \qquad -8x$$

$$0 = 2x^2-10x-3$$

$a=2,\ b=-10,\ c=-3$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{10 \pm \sqrt{(-10)^2-4\times2\times-3}}{4}$$

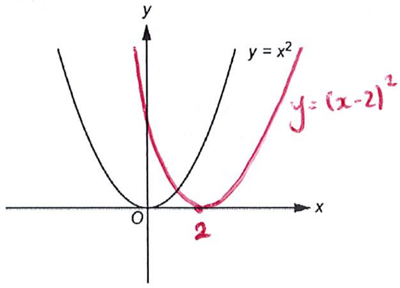
$(+) = 5.28$

$(-) = -0.28$

$x = 5.28$ or $x = -0.28$

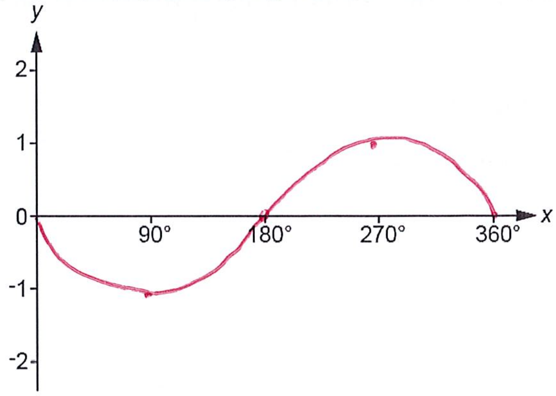
Transformations of Graphs

a) This is a sketch of the graph of $y = x^2$.



Sketch the graph of $y = (x - 2)^2$ on the same axes.

b) Sketch the graph of $y = -\sin x$ for $0° \leq x \leq 360°$.



Algebraic Proof

a) Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8.

Consecutive odd numbers: $2n - 1$, $2n + 1$

$(2n+1)^2 - (2n-1)^2 = (4n^2 + 4n + 1) - (4n^2 - 4n + 1) = 8n$

Therefore it is a multiple of 8.

b) Prove that $(2x + 1)(3x + 2) + x(3x + 5) + 2$ is a perfect square.

	$2x$	$+1$
$3x$	$6x^2$	$+3x$
$+2$	$+4x$	$+2$

$6x^2 + 7x + 2 + 3x^2 + 5x + 2$

$= 9x^2 + 12x + 4 = (3x+2)^2$, which is a perfect square.

END OF QUESTIONS