

NUMBER

Multiply & Divide Decimals

Evaluate:

a) 3.4×1.927

$$\begin{array}{r} 34 \\ 10 \end{array} \times \begin{array}{r} 1927 \\ 1000 \end{array}$$

$$\frac{65118}{10,000}$$

$$\begin{array}{r|l} 4 & 4,000 \quad 3,600 \quad 80 \quad 28 \\ 30 & 30,000 \quad 27,000 \quad 600 \quad 210 \\ \hline & 3000 \quad 900 \quad 20 \quad 7 \end{array}$$

.....
6.5118

b) $57.3 \div 8$

$$\begin{array}{r} 07.1625 \\ 8 \overline{) 57.3} \\ \underline{-0} \\ 57 \\ \underline{-56} \\ 13 \\ \underline{-8} \\ 50 \\ \underline{-48} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

.....
7.1625

c) $79.5 \div 0.15$

$$\frac{79.5}{0.15} = \frac{7950}{15}$$

$$\begin{array}{r} 0530 \\ 15 \overline{) 7950} \end{array}$$

.....
530

Product Rule for Counting

Awatif has four cards with the numbers 1, 3, 5 and 8. She uses the four cards to create a four-digit number.

a) In how many ways can this be done?

$$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$$

.....
24

She then uses the cards to create a three-digit even number.

b) How many different three-digit even numbers can be created?

$$\begin{array}{c} \underline{3} \times \underline{2} \times \underline{1} \\ \phantom{\underline{3} \times \underline{2} \times} \uparrow \\ \phantom{\underline{3} \times \underline{2} \times} 8 \end{array}$$

.....
6

Estimations

Lei is in a class of 28 pupils, 3 of whom are left-handed.

There are 1250 pupils in the school.

- a) Use this information to estimate how many pupils in the school are left-handed.

$$\frac{3}{28} \times 1250 \approx \frac{3}{30} \times 1250 = \frac{1}{10} \times 1250$$

.....125.....

- b) Is your solution to (a) likely to be an overestimate or an underestimate?
Explain your reasoning.

Underestimate. To estimate I rounded the denominator up, which decreased the proportion of left-handed people from $\frac{3}{28}$ to $\frac{3}{30}$.

Laws of Indices

Write as a single power:

a) $23^{52} \times 23^{-13}$

..... 23^{39}

b) $m^{23} \div m^7$

..... m^{16}

c) $(125^6)^{15}$

..... 125^{90}

Write as a single power of 2:

d) $\frac{32^3 \times 0.125}{0.25}$

$$= \frac{(2^5)^3 \times 2^{-3}}{2^{-2}} = \frac{2^{15} \times 2^{-3}}{2^{-2}} = \frac{2^{12}}{2^{-2}}$$

..... 2^{14}

Negative and Fractional Indices

Evaluate:

a) 7^{-2}

$$= \frac{1}{7^2}$$

$$\frac{1}{49}$$

b) $27^{\frac{1}{3}}$

$$= \sqrt[3]{27}$$

$$3$$

c) $32^{\frac{3}{5}}$

$$= (\sqrt[5]{32})^3$$

$$8$$

d) Without using a calculator, show clearly that $64^{\frac{2}{3}}$ is equal to 16.

$$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$$

e) Write $\sqrt[8]{27 \times 3}$ in the form 3^k , where k is a fraction in its simplest form.

$$\sqrt[8]{3^3 \times 3} = \sqrt[8]{3^4} = (3^4)^{\frac{1}{8}} = 3^{\frac{4}{8}} = 3^{\frac{1}{2}}$$

$$\frac{1}{2}$$

Product of Prime Factors

a) Write 120 as a product of its prime factors.

$$\begin{aligned} 120 &= 12 \times 10 \\ &= 4 \times 3 \times 2 \times 5 \\ &= 2 \times 2 \times 3 \times 2 \times 5 = 2 \times 2 \times 2 \times 3 \times 5 \end{aligned}$$

$$2^3 \times 3 \times 5$$

b) $177147 = 3^{11}$

Write 177 417 000 000 as a product of its prime factors.

$$\begin{aligned} &= 3^{11} \times 1000000 \\ &= 3^{11} \times (10)^6 \\ &= 3^{11} \times 2^6 \times 5^6 \end{aligned}$$

$$2^6 \times 3^{11} \times 5^6$$

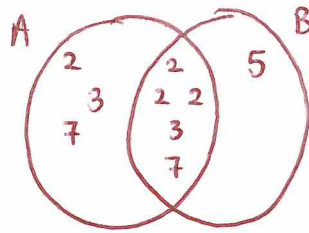
Highest Common Factor

- a) Two numbers, A and B, are written as a product of prime factors.

$$A = 2^4 \times 3^2 \times 7^2$$

$$B = 2^3 \times 3 \times 5 \times 7$$

Find the Highest Common Factor (HCF) of A and B.



$$2 \times 2 \times 2 \times 3 \times 7$$

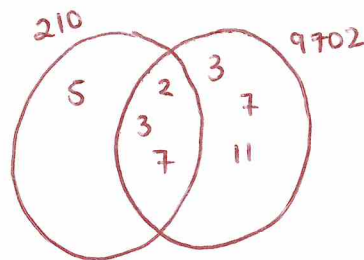
168

- b) The number 9702 can be written as $2 \times 3^2 \times 7^2 \times 11$.

Find the Highest Common Factor (HCF) of 210 and 9702.

$$210 = 21 \times 10$$

$$= 3 \times 7 \times 2 \times 5$$



$$2 \times 3 \times 7$$

42

Lowest Common Multiple

- a) At a railway station, trains are either eastbound or westbound.

An eastbound train leaves the station every 25 minutes.

A westbound train leaves the station every 45 minutes.

An eastbound train and a westbound train both leave the station at 8 am.

When is the next time that two trains leave the station together?

$$25, 50, 75, 100, 125, 150, 175, 200, 225$$

$$45, 90, 135, 180, 225$$

$$225m = 3h45m$$

$$8am + 3h45m$$

11:45am

- b) Find the Lowest Common Multiple (LCM) of 180 and 504.

$$180 = 18 \times 10$$

$$= 2 \times 3 \times 3 \times 2 \times 5 = 2 \times 2 \times 3 \times 3 \times 5$$

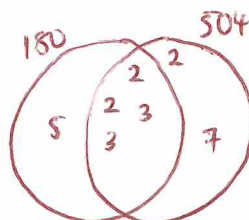
$$504 = 2 \times 252$$

$$= 2 \times 2 \times 126$$

$$= 2 \times 2 \times 2 \times 63$$

$$= 2 \times 2 \times 2 \times 3 \times 21$$

$$= 2 \times 2 \times 2 \times 3 \times 3 \times 7$$



$$2 \times 2 \times 3 \times 3 \times 5 \times 2 \times 7$$

2520

Standard Form Conversions

The table shows the mass of apples produced in some countries in 2014.

All values are given to 3 significant figures.

Country	Mass of Apples (tonnes)
Denmark	3.54×10^4
France	1.89×10^6
Italy	2.45×10^6
Poland	3.20×10^6
UK	4.04×10^5

Poland produced 3 195 300 tonnes of apples.

The UK produced 404 200 tonnes of apples.

a) Use this information to complete the table. Write each number correct to 3 significant figures.

b) Write down the mass, in kilograms, of apples produced in Denmark. Give your answer in standard form. ~~in standard form.~~ *in standard form.*

$$\dots 3.54 \times 10^7 \text{ kg}$$

Standard Form Calculations

a) Work out $5 \times 10^4 - 1.6 \times 10^3$
Give your answer in standard form.

$$5 \times 10^4 - 0.16 \times 10^4$$

$$\dots 4.84 \times 10^4$$

b) Work out $(2 \times 10^3) \times (6 \times 10^4)$
Give your answer in standard form.

$$= 12 \times 10^7$$

$$\dots 1.2 \times 10^8$$

c) Use the formula to find the value of v when $G = 6.67 \times 10^{-11}$, $M = 5.97 \times 10^{24}$,
and $r = 6.4 \times 10^6$.

$$v = \sqrt{\frac{2GM}{r}}$$

$$\dots 11,155.14$$

$$\approx 1.12 \times 10^4$$

Simplifying Surds

a) Without using a calculator, show that $\sqrt{20} = 2\sqrt{5}$.

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \times \sqrt{5} \\ &= 2 \times \sqrt{5} = 2\sqrt{5}\end{aligned}$$

b) Show that $\sqrt{396}$ can be written as $6\sqrt{11}$.

$$\begin{aligned}\sqrt{396} &= \sqrt{36} \times \sqrt{11} \\ &= 6 \times \sqrt{11} = 6\sqrt{11}\end{aligned}$$

c) Simplify $3\sqrt{2} \times \sqrt{6}$

$$\begin{aligned}3\sqrt{2} \times \sqrt{6} \\ &= 3 \times \sqrt{2} \times \sqrt{2} \times \sqrt{3} \\ &= 3 \times 2 \times \sqrt{3}\end{aligned}$$

$6\sqrt{3}$

Surds Calculations

a) Simplify $\sqrt{50} + \sqrt{2}$

$$\begin{aligned}\sqrt{50} &= \sqrt{25} \times \sqrt{2} \\ &= 5 \times \sqrt{2} = 5\sqrt{2}\end{aligned}$$

$$5\sqrt{2} + \sqrt{2}$$

$6\sqrt{2}$

b) Write $\sqrt{12} + \sqrt{75}$ in the form $k\sqrt{3}$

$$\begin{aligned}\sqrt{12} &= \sqrt{4} \times \sqrt{3} = \\ &= 2 \times \sqrt{3} = 2\sqrt{3}\end{aligned}$$

$$2\sqrt{3} + 5\sqrt{3}$$

$$\begin{aligned}\sqrt{75} &= \sqrt{25} \times \sqrt{3} \\ &= 5 \times \sqrt{3} = 5\sqrt{3}\end{aligned}$$

$7\sqrt{3}$

c) Write $(1 + \sqrt{3})^2$ in the form $a + b\sqrt{3}$.

$$= (1 + \sqrt{3})(1 + \sqrt{3})$$

$$\begin{array}{r|l} +\sqrt{3} & \sqrt{3} \quad 3 \\ 1 & 1 \quad \sqrt{3} \\ \hline & 1 \quad +\sqrt{3} \end{array}$$

$$= 1 + 3 + \sqrt{3} + \sqrt{3}$$

$4 + 2\sqrt{3}$

Rationalising Fractional Surds

a) Simplify $\frac{6}{\sqrt{2}}$

$$\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

.....
 $3\sqrt{2}$

b) Without using a calculator, show that $\frac{4+2\sqrt{2}}{2-\sqrt{2}}$ can be simplified to $6+4\sqrt{2}$.

$$\frac{4+2\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{12+8\sqrt{2}}{2} = 6+4\sqrt{2}$$

$$\begin{array}{r|l} +\sqrt{2} & 4\sqrt{2} \quad 4 \\ \hline 2 & 8 \quad 4\sqrt{2} \\ \hline & 4 \quad +2\sqrt{2} \end{array}$$

$$\begin{array}{r|l} +\sqrt{2} & 2\sqrt{2} \quad -2 \\ \hline 2 & 4 \quad -2\sqrt{2} \\ \hline & 2 \quad -\sqrt{2} \end{array}$$

Fraction Calculations

Calculate:

a) $2\frac{3}{8} \div 1\frac{1}{18}$

Give your answer as a mixed number in its simplest form.

$$\frac{19}{8} \div \frac{19}{18} = \frac{19}{8} \times \frac{18}{19} = \frac{9}{4} = 2\frac{1}{4}$$

.....
 $2\frac{1}{4}$

b) $\frac{2}{15} \times \frac{15}{22}$

Give your answer in its lowest terms.

$$\frac{2}{15} \times \frac{15}{22} = \frac{1}{11}$$

.....
 $\frac{1}{11}$

Recurring Decimals

- a) Write $\frac{5}{11}$ as a recurring decimal.

$$\begin{array}{r} 0.4545\ldots \\ 11 \overline{) 5.000000} \end{array}$$

..... $0.\dot{4}\dot{5}$

- b) Convert $0.3\dot{6}$ to a fraction.
Give your answer in its simplest form.

$$\begin{aligned} x &= 0.3\dot{6} \\ 10x &= 3.\dot{6} \\ 100x &= 36.\dot{6} \end{aligned}$$

$$100x - 10x$$

$$90x = 33$$

$$x = \frac{33}{90} = \frac{11}{30}$$

..... $\frac{11}{30}$

Percentage Calculations

- a) Between January 2015 and January 2016 the price of diesel increased from £1.12 per litre to £1.19 per litre.
Calculate the percentage increase in price.

$$\frac{1.19}{1.12} = 1.0625 \quad \uparrow \quad 106.25\%$$

..... 6.25%

- b) A shop has a sale that offers 20% of all prices.
On the final day, they reduce all sale prices by 25%.
Alex buys a hairdryer on the final day.
Work out the overall percentage reduction on the price of the hairdryer.

$$\begin{aligned} &x \times 0.8 \times 0.75 \\ &= x \times 0.6 \\ &\quad \uparrow \\ &\quad 60\% \end{aligned}$$

..... 40%

Reverse Percentages

- a) When water freezes into ice its volume increases by 9%.
What volume of water freezes to make 1962cm^3 of ice?

$$109\% = 1962$$

$$\frac{1962}{1.09} = 1800$$

$$\underline{\underline{1800\text{cm}^3}}$$

- b) Amrit's income is 32% more than Bethan's income.
Amrit and Bethan's combined income is £54 868.
Calculate Amrit's income.

$$b + 1.32b = 54868$$

$$2.32b = 54868$$

$$b = \frac{54868}{2.32} = 23650$$

$$23650 \times 1.32$$

$$\underline{\underline{£31,218}}$$

Error Intervals

- a) A number, x , rounded to one decimal place, is 4.7.
Write the error interval for x .

$$\underline{\underline{4.65 \leq x < 4.75}}$$

- b) A number, y , rounded to two decimal places, is 4.13.
Write the error interval for y .

$$\underline{\underline{4.125 \leq y < 4.135}}$$

- c) A number, z , rounded to two significant figures, is 4700.
Write the error interval for z .

$$\underline{\underline{4650 \leq z < 4750}}$$

Calculating with Bounds

- a) Some boxes are to be loaded into a van.

Each box measures exactly 40 cm by 30 cm by 50 cm.

Each box weighs 40 kg, correct to the nearest kilogram.

The loading space in the van measures exactly 110 cm by 90 cm by 180 cm.

The maximum total weight of the boxes that can be loaded into the van is 890 kg, correct to the nearest 10 kilograms.

Work out the maximum number of boxes that can be loaded into the van without exceeding the weight limit.

Show clearly how you worked out your answer.

$$39.5 \leq \text{Box} < 40.5 \qquad \frac{885}{40.5} = 21.85185185\dots \quad 21 \text{ boxes, by weight.}$$

$$885 \leq \text{Load} < 895$$

$$\text{Volume (van)} = 110 \times 90 \times 180 = 1\,782\,000$$

$$\text{Volume (box)} = 40 \times 30 \times 50 = 60,000 \qquad = 29.7 \text{ boxes, in terms of volume}$$

$$\left. \begin{array}{l} 180 \times 90 \times 110 \\ 30 \times 40 \times 50 \end{array} \right\} 6 \times 2 \times 2 = 24 \text{ boxes, arranging boxes}$$

.....
21

- b) Claudine cycled a distance of 53 km in 2.7 hours.

The distance is measured correct to the nearest km.

The time is given correct to 1 decimal place.

Calculate the lower and upper bounds of her average speed.

Give your answers correct to 2 decimal places.

$$52.5 \leq \text{distance} < 53.5 \text{ km}$$

$$2.65 \leq \text{time} < 2.75 \text{ hrs}$$

$$\text{speed}_{\max} = \frac{\text{distance}_{\max}}{\text{time}_{\min}} = \frac{53.5}{2.65} = 20.188679245$$

$$\text{speed}_{\min} = \frac{\text{distance}_{\min}}{\text{time}_{\max}} = \frac{52.5}{2.75} = 19.090909\dots$$

lower bound =19.09..... km / h

upper bound =20.19..... km / h

END OF QUESTIONS