TRIGONOMETRY

Pythagoras' Theorem

a) A triangle has sides of length 23.8cm, 31.2cm and 39.6cm.
Is this a right-angled triangle?
Show how you decide.

$23.8^2 = 566.44$

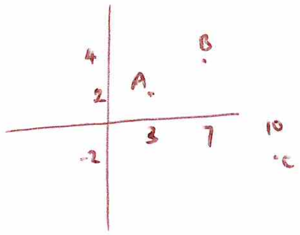
$31.2^2 = 973.44$

$39.6^2 = 1568.16$

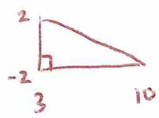
$23.8^2 + 31.2^2 \neq 39.6^2$

Not right-angled.

b) A is the point (3, 2), B is the point (7, 4) and C is the point (10, −2).
Calculate the length of the hypotenuse of triangle ABC.



AC is the hypotenuse

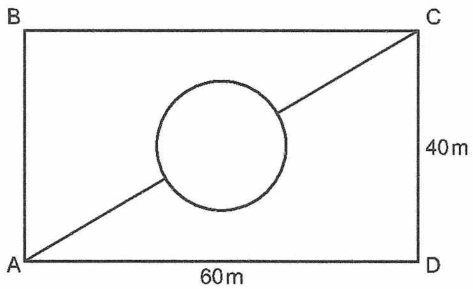


$4^2 + 7^2 = 16 + 49 = 65$

$\sqrt{65} = 8.062257748$

.........8.1.........

c) The rectangle ABCD represents a park.



Not to scale

The lines show all the paths in the park.
The circular path is in the centre of the rectangle and has a diameter of 10 m.

Calculate the shortest distance from A to C across the park, using only the paths shown.

$AC = \sqrt{40^2 + 60^2} = 72.111025509$

$- 10$ (diameter)

Straight Parts $= 62.111025509$

Circular path $= \pi \times 10 = 31.415926536$

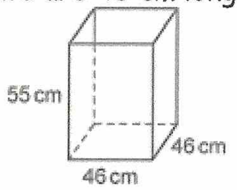
Curved half $=$ ANS $\div 2 = 15.707963268$

62.111025509
$+ 15.707963268$
77.818988777

.........77.82.........m

Pythagoras' Theorem in 3D

a) Alvin has a crate in the shape of a cuboid.
The crate is open at the top.
The internal dimensions of the crate are 46 cm long by 46 cm wide by 55 cm high.



Alvin has a stick of length 95 cm.
Alvin places the stick in the crate so that the shortest possible length extends out above the top of the crate.
Calculate the length of the stick that extends out of the crate.

Space diagonal = $\sqrt{55^2+46^2+46^2} = 85.188027328$

$95 - 85.188027328 = 9.811972672$

..........9.8cm..........

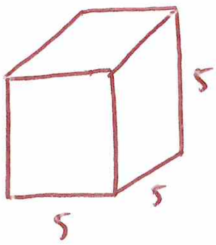
b) The length of the longest diagonal of a cube is 25 cm.
Calculate the total surface area of the cube.

$\sqrt{x^2+x^2+x^2} = 25$

$3x^2 = 75$

$x^2 = 25$

$x = \pm 5$

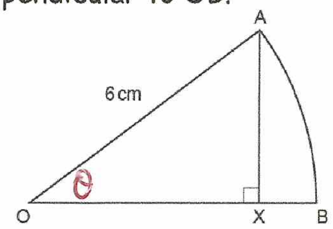


$6 \times 5^2 = 150$

..........150..........cm^2

Trigonometric Ratios for Side Lengths

a) OAB is a sector of a circle, centre O.
OA = 6cm and AX is perpendicular to OB.



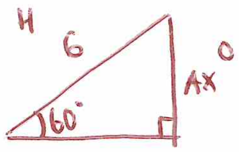
Not to scale

The area of the sector is 6π cm^2.
Show that AX = $3\sqrt{3}$cm.

$$\text{Area} = \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times \pi \times 6^2$$

$$6\pi = \frac{\theta}{360} \times 36\pi$$

$$\frac{\theta}{360} = \frac{1}{6} \qquad \theta = 60°.$$

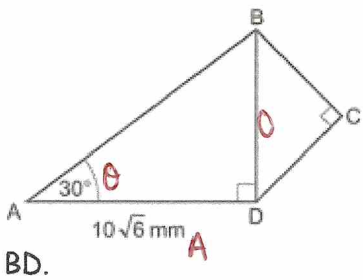


$$\sin 60 = \frac{O}{H}$$

$$\frac{\sqrt{3}}{2} = \frac{AX}{6}$$

$$\frac{6\sqrt{3}}{2} = AX = 3\sqrt{3}.$$

b) ABD and BCD are right-angled.
BC = CD.
AD = $10\sqrt{6}$ mm.
Angle BAD = 30°.

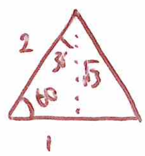


Not to scale

Calculate the length of BD.

$$\tan \theta = \frac{O}{A}$$

$$\tan 30 = \frac{BD}{10\sqrt{6}}$$



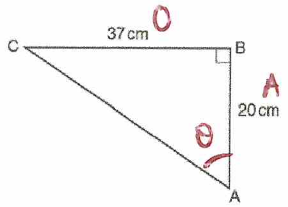
$$\tan 30 = \frac{1}{\sqrt{3}}$$

$$10\sqrt{6} \times \tan 30 = BD \qquad 10\sqrt{6} \times \frac{1}{\sqrt{3}} = 10\sqrt{2}$$

$10\sqrt{2}$ ....

Trigonometric Ratios for Angles

a) ABC is a right-angled triangle.
AB = 20cm and BC = 37cm.



Not to scale

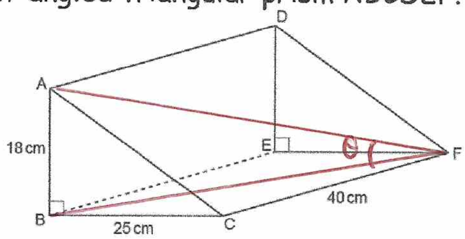
Calculate angle BAC.

$\tan\theta = \frac{37}{20}$

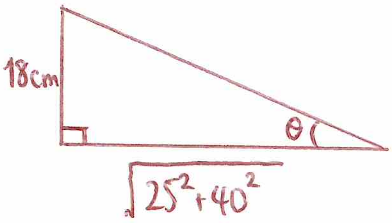
$\theta = \tan^{-1}\left(\frac{37}{20}\right) = 61.606980579$

........61.6°........

b) The diagram shows a right-angled triangular prism ABCDEF.



Calculate the angle AFB.



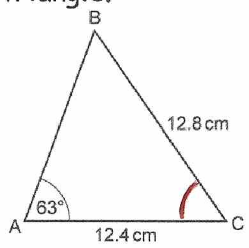
$\tan\theta = \frac{18}{\sqrt{25^2+40^2}}$

$\theta = \tan^{-1}\left(\frac{18}{\sqrt{25^2+40^2}}\right) = 20.886316203$

........20.9........°

Sine Rule

a) Calculate angle ACB in this triangle.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

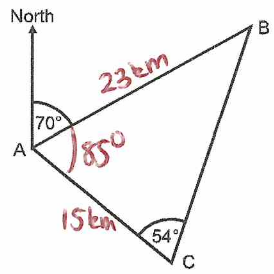
$$\frac{\sin 63}{12.8} = \frac{\sin B}{12.4}$$

$$\frac{12.4 \times \sin 63}{12.8} = \sin B \qquad B = \sin^{-1}\left(\frac{12.4 \times \sin 63}{12.8}\right) = 59.673552453^\circ$$

$$C = 180 - (\text{ANS} + 63)$$
$$= 27.326447547$$

.......27.3....... °

b) The diagram shows the positions of three hills, A, B and C.



B is 23 km from A on a bearing of 070°.
C is 15 km from A.
Angle ACB = 54°.

Find the bearing of C from A.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$23^2 = b^2 + 15^2 - 2 \times 15 \times b \times \cos 54$$

$$529 = b^2 + 225 - 30\cos 54\, b$$

$$0 = b^2 - 30\cos 54\, b - 304$$

$$b = \frac{30\cos 54 \pm \sqrt{(30\cos 54)^2 - 4 \times 1 \times -304}}{2}$$

$= (+)\ 26.326524250$

$(-)\ -8.69296681$ $\qquad BC = 26.326524250$ km

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{23^2 + 15^2 - 26.326524250^2}{2 \times 23 \times 15}$$

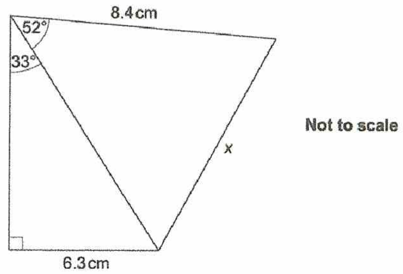
$$A = \cos^{-1}(\text{ANS}) = 84.935258773$$

$$70 + 85 = 155$$

.......155....... °

Cosine Rule

a) Calculate x.



$$x^2 = 8.4^2 + \left(\frac{6.3}{\sin 33}\right)^2 - 2 \times 8.4 \times \frac{6.3}{\sin 33} \times \cos 52$$

$$\sin 33 = \frac{6.3}{H}$$

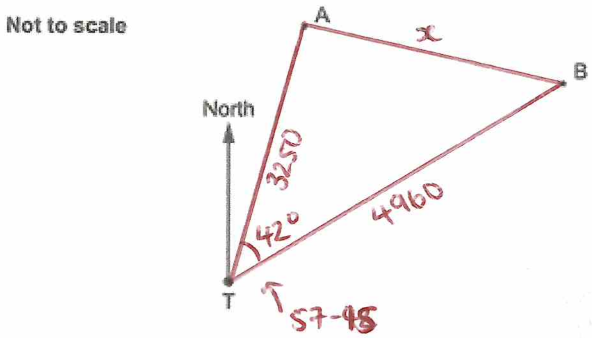
$$x = \sqrt{ANS} = 9.204372207$$

$$H = \frac{6.3}{\sin 33}$$

.........9.2.........cm

b) T is a radar tower.
A and B are two aircraft.

At 3pm, aircraft A is 3250 km from T on a bearing of 015°, and aircraft B is 4960 km from T on a bearing of 057°.



Calculate the distance that was between aircraft A and aircraft B at 3pm.

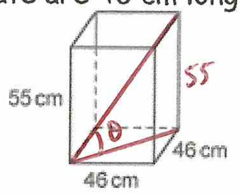
$$x^2 = 3250^2 + 4960^2 - 2 \times 3520 \times 4960 \times \cos 42$$

$$x = \sqrt{ANS} = 3347.403594819$$

.........3347.4.........km

3D Trigonometry

a) Alvin has a crate in the shape of a cuboid.
The crate is open at the top.
The internal dimensions of the crate are 46 cm long by 46 cm wide by 55 cm high.



Alvin has a stick of length 95 cm.
Alvin places the stick in the crate so that the shortest possible length extends out above the top of the crate.
Calculate the angle that the stick makes with the base of the crate.

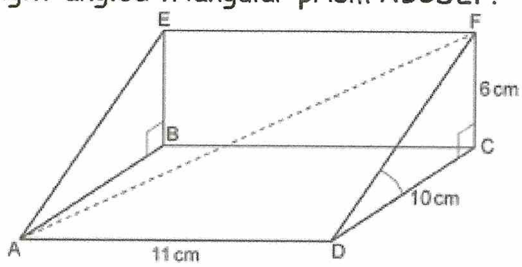
~~Space di~~ Base diagonal $= \sqrt{46^2+46^2}$

$\tan\theta = \dfrac{55}{\sqrt{46^2+46^2}}$

$\theta = \tan^{-1}\left(\dfrac{55}{\sqrt{46^2+46^2}}\right) = 40.212974639$

.......40.2........°

b) The diagram shows a right-angled triangular prism ABCDEF.



Length AD = 11 cm, length CD = 10 cm and length CF = 6 cm.
Use trigonometry to show that angle FDC = 31°, correct to the nearest degree.

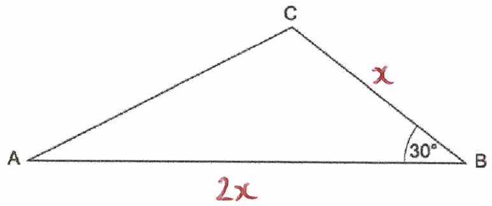
$\tan\theta = \dfrac{6}{10}$ $\quad \theta = \tan^{-1}\left(\dfrac{6}{10}\right) = 30.963756532$

$= 31°$ to the nearest degree

Area of a Triangle

Triangle ABC has area 40 cm^2.

AB = 2BC.



Not to scale

Work out the length of BC.

Give your answer as a surd in its simplest form.

$\frac{1}{2}ab\sin C = \text{Area}$

$\frac{1}{2} \times x \times 2x \times \sin 30 = 40$

$\frac{x^2}{2} = 40$

$x^2 = 80$

$x = \sqrt{80} = \sqrt{16} \times \sqrt{5}$

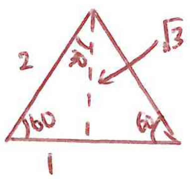
$4\sqrt{5}$

......$4\sqrt{5}$......cm

Exact Trigonometric Values

Write down the exact value of:

a) tan 60°



$\tan\theta = \frac{O}{A}$

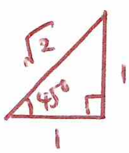
......$\sqrt{3}$......

b) cos 30°

$\cos\theta = \frac{A}{H}$

......$\frac{\sqrt{3}}{2}$......

c) sin 45°



$\sin\theta = \frac{O}{H}$

......$\frac{1}{\sqrt{2}}$......

d) One solution to the equation $4\sin x = k$ is $x = 60°$.

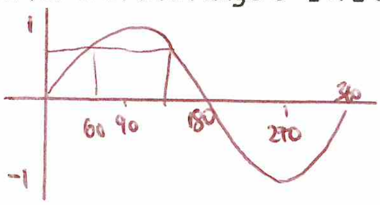
i) Find the value of k.

$k = 4 \times \sin 60$

$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

......$2\sqrt{3}$......

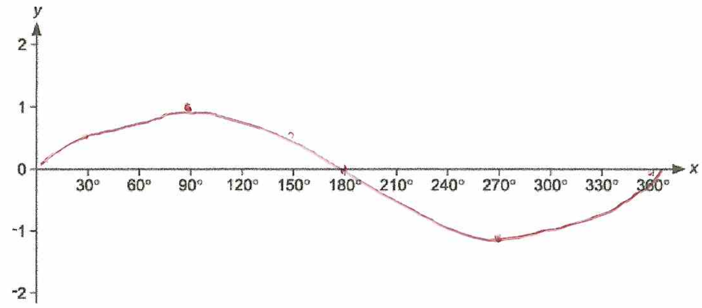
ii) Find another solution for x in the range $0° \le x \le 360°$



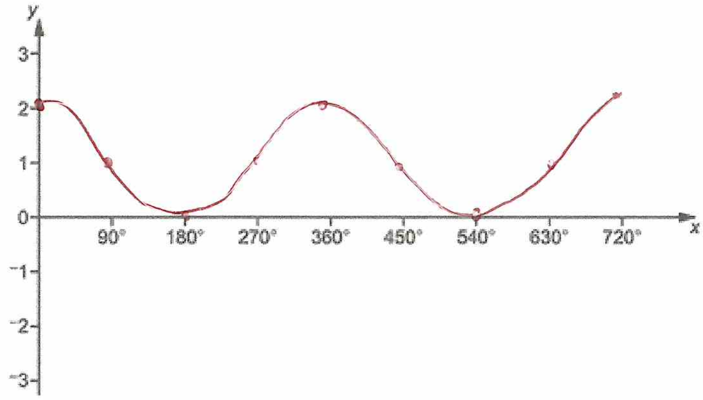
......120°......

Trigonometric Graphs

a) Sketch the graph of $y = \sin x$ for $0° \le x \le 360°$

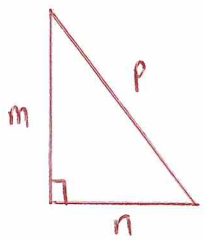


b) Sketch the graph of $y = \cos x + 1$ for $0° \le x \le 720°$



Trigonometric Proof

The lengths of the sides of a right-angled triangle are all integers.
Prove that if the lengths of the two shortest sides are even, then the length of the third side must also be even.



m is even $\quad m^2$ is even

n is even $\quad n^2$ is even

m^2+n^2 is even

$m^2+n^2=p^2$

p^2 is even

p is even.

END OF QUESTIONS