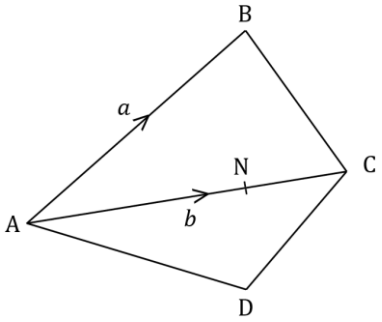
a)

In the diagram, $\overrightarrow{DC} = \frac{1}{2}\overrightarrow{AB}$.

N is on the line AC such that $AN : NC$ is $2 : 1$.

Prove that BND is a straight line.

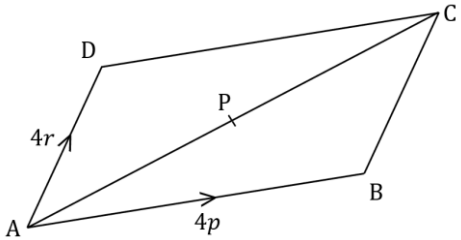


What makes BND a straight line?	The vectors $\overrightarrow{BN}$ and $\overrightarrow{ND}$ are parallel to one another, and share a common point, N.
How can we write $\overrightarrow{BN}$ in terms of a and b?	$\overrightarrow{BN} = \overrightarrow{BA} + \overrightarrow{AN}$ $\overrightarrow{BN} = -a + \frac{2}{3}\overrightarrow{AC} = -a + \frac{2}{3}b = \frac{2}{3}b - a$
How can we write $\overrightarrow{ND}$ in terms of a and b?	$\overrightarrow{ND} = \overrightarrow{NC} + \overrightarrow{CD}$ $\overrightarrow{ND} = \frac{1}{3}\overrightarrow{AC} + \overrightarrow{CD} = \frac{1}{3}b - \frac{1}{2}a$
What can we say about the vectors $\overrightarrow{BN}$ and $\overrightarrow{ND}$?	$\overrightarrow{BN} = \frac{2}{3}b - a$ $\quad$ $\overrightarrow{ND} = \frac{1}{3}b - \frac{1}{2}a$ $\overrightarrow{BN} = 2\overrightarrow{ND}$, so $\overrightarrow{BN}$ and $\overrightarrow{ND}$ are parallel.
What can we say about the line BND?	Because $\overrightarrow{BN}$ and $\overrightarrow{ND}$ are parallel, and share a common point, N, we can say that BND is a straight line.

b)

In the diagram, ABCD is a parallelogram, and P is the midpoint of the line AC.

Prove that BPD is a straight line.

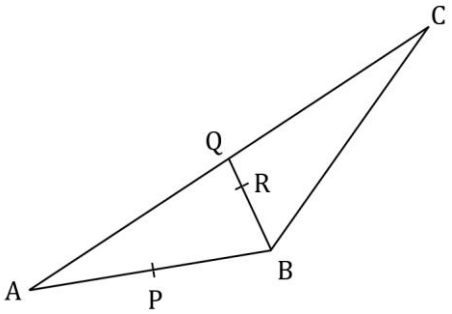


What makes BPD a straight line?	The vectors $\overrightarrow{BP}$ and $\overrightarrow{PD}$ are parallel to one another, and share a common point, P.
How can we write $\overrightarrow{BP}$ in terms of a and b?	$\overrightarrow{BP} = \overrightarrow{BA} + \overrightarrow{AP}$ $\overrightarrow{BP} = -4p + \frac{1}{2}\overrightarrow{AC} = -4p + \frac{1}{2}(4p + 4r)$ $= -4p + 2p + 2r = 2r - 2p$
How can we write $\overrightarrow{PD}$ in terms of a and b?	$\overrightarrow{PD} = \overrightarrow{PC} + \overrightarrow{CD}$ $\overrightarrow{PD} = \frac{1}{2}\overrightarrow{AC} + (-4p) = \frac{1}{2}(4p + 4r) - 4p$ $= 2p + 2r - 4p = 2r - 2p$
What can we say about the vectors $\overrightarrow{BP}$ and $\overrightarrow{PD}$?	
What can we say about the line BPD?	

c)

In the diagram, $\overrightarrow{AB} = a$, and $\overrightarrow{BC} = b$.
P is the mid-point of AB and Q is the mid-point of AC.
R is a point on the line BQ such that $BR : RQ$ is $1 : 2$.

Prove that PRC is a straight line.



What makes PRC a straight line?	The vectors $\overrightarrow{PR}$ and $\overrightarrow{RC}$ are parallel to one another, and share a common point, R.
How can we write $\overrightarrow{PR}$ in terms of a and b?	$\overrightarrow{PR} = \overrightarrow{PB} + \overrightarrow{BR}$ $\overrightarrow{PR} = \frac{1}{2}\overrightarrow{AB} + \frac{2}{3}\overrightarrow{BQ} = \frac{1}{2}a + \frac{2}{3}\left(-\frac{1}{2}a + \frac{1}{2}b\right)$ $= \frac{1}{2}a - \frac{1}{3}a + \frac{1}{3}b = \frac{1}{6}a + \frac{1}{3}b$
How can we write $\overrightarrow{RC}$ in terms of a and b?	
What can we say about the vectors $\overrightarrow{PR}$ and $\overrightarrow{RC}$?	
What can we say about the line PRC?	

d)

In the diagram, ABCD is a parallelogram.
E is the point on the extended line such that $CD : DE = 1 : 2$.
P is the point on AD such that $AP : PD = 1 : 2$

Prove that BPE is a straight line.