

a)

Differentiate $f(x) = 4x^2$ from first principles.

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{f(x + \delta) - f(x)}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{4(x + \delta)^2 - 4x^2}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{4x^2 + 8\delta x + 4\delta^2 - 4x^2}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{8\delta x + 4\delta^2}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} 8x + 4\delta$$

$$f'(x) = 8x$$

b)

Differentiate $f(x) = x^3$ from first principles.

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{f(x + \delta) - f(x)}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{(x + \delta)^3 - x^3}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{x^3 + 3\delta x^2 + 3\delta^2 x + \delta^3 - x^3}{\delta}$$

c)

Differentiate $f(x) = \frac{x^2}{2}$ from first principles.

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{f(x + \delta) - f(x)}{\delta}$$

$$f'(x) = \lim_{\delta \rightarrow 0} \frac{\frac{(x + \delta)^2}{2} - \frac{x^2}{2}}{\delta}$$

d)

Differentiate $f(x) = x^2 - x$ from first principles.