

a)

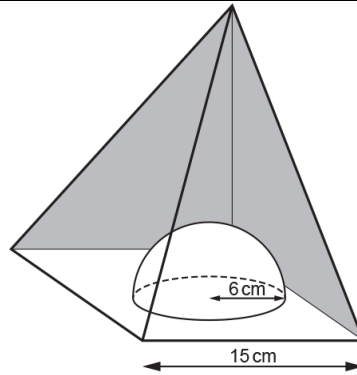
An ornament is made from a solid glass square-based pyramid.

The base has side length 15cm.

A hemisphere with radius 6cm is cut out of the base of the pyramid.

This reduces the volume of glass contained in the ornament by 30%.

Calculate the perpendicular height of the pyramid.



b)

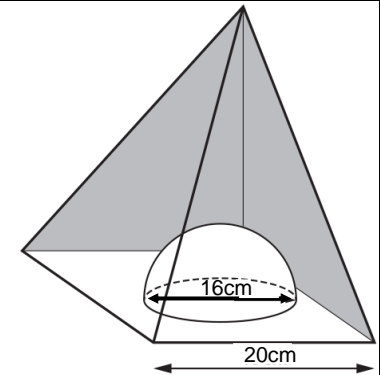
An ornament is made from a solid glass square-based pyramid.

The base has side length 20cm.

A hemisphere with diameter 16cm is cut out of the base of the pyramid.

This reduces the volume of glass contained in ornament by 25%.

Calculate the perpendicular height of the pyramid.



What is the volume of the 'full' pyramid?	$V_{pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perp. height}$ $= \frac{1}{3} \times 15^2 \times x = 75x$
What is the volume of the hemisphere?	$V_{sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times 6^3 = 288\pi$ $V_{hemisphere} = \frac{288\pi}{2} = 144\pi$
What is the relationship between the two volumes?	$V_{hemisphere} = 30\% \text{ of } V_{pyramid}$ $144\pi = \frac{3}{10} \times 75x$ $144\pi = \frac{225x}{10}$
What is the perpendicular height, $x$ ?	$x = \frac{144\pi \times 10}{225} = 20.10619298297 \dots$ <p style="text-align: center;">20.11cm</p>

What is the volume of the 'full' pyramid?	$V_{pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perp. height}$ $= \frac{1}{3} \times 20^2 \times x = \frac{400}{3}x$
What is the volume of the hemisphere?	$V_{sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times 8^3 = \frac{2048\pi}{3}$ $V_{hemisphere} = \frac{2048\pi}{3} \div 2 = \frac{1024\pi}{3}$
What is the relationship between the two volumes?	$V_{hemisphere} = 25\% \text{ of } V_{pyramid}$ $\frac{1024\pi}{3} = \frac{1}{4} \times \frac{400}{3}x$ $\frac{1024\pi}{3} = \frac{100x}{3}$
What is the perpendicular height, $x$ ?	

c)

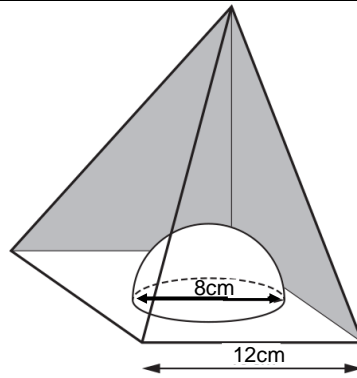
An ornament is made from a solid glass square-based pyramid.

The base has side length 12cm.

A hemisphere with diameter 8cm is cut out of the base of the pyramid.

This reduces the volume of glass contained in ornament by 40%.

Calculate the perpendicular height of the pyramid.



d)

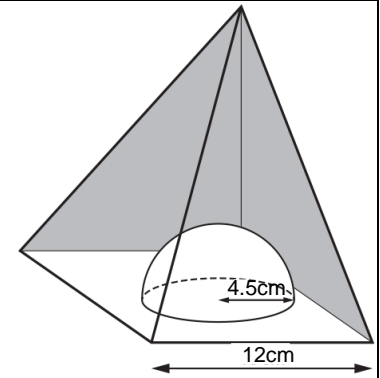
An ornament is made from a solid glass square-based pyramid.

The base has side length 12cm.

A hemisphere with radius 4.5cm is cut out of the base of the pyramid.

This reduces the volume of glass contained in the ornament by 20%.

Calculate the perpendicular height of the pyramid.



What is the volume of the 'full' pyramid?	$V_{pyramid} = \frac{1}{3} \times \text{area of base} \times \text{perp. height}$ $= \frac{1}{3} \times 12^2 \times x = 48x$
What is the volume of the hemisphere?	$V_{sphere} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \pi \times 4^3 = \frac{256\pi}{3}$ $V_{hemisphere} = \frac{256\pi}{3} \div 2 = \frac{128\pi}{3}$
What is the relationship between the two volumes?	
What is the perpendicular height, $x$ ?	