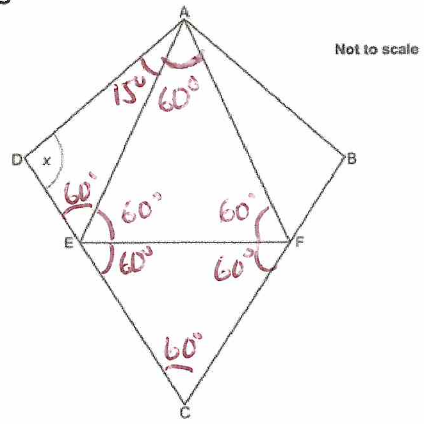
GEOMETRY

Triangles and Quadrilaterals

The diagram shows a kite, ABCD.
AFE and CEF are equilateral triangles.



Write down a mathematical name for quadrilateral AFCE.

Rhombus

The ratio of angle DAE : angle EAF = 1 : 4.

15 : 60°

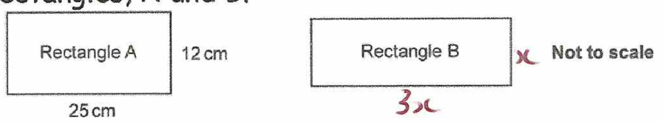
Work out angle x.
Write on the diagram the values of any other angles you use in your working.

$180 - (60 + 15) = 105$

105°

Area of 2D Shapes

The diagram shows two rectangles, A and B.



Rectangle A has a width of 25 cm and a height of 12 cm.
The width of rectangle B is three times the height of rectangle B.

The area of rectangle A is equal to the area of rectangle B.
Find the perimeter of rectangle B.

Area (A) $= 12 \times 25 = 300\text{cm}^2$

Area (B) $= 3x \times x = 3x^2$

$3x^2 = 300$

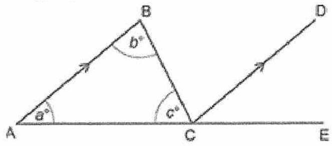
$x^2 = 100 \quad x = 10$

$10 + 30 + 10 + 30$

80 cm

Angles in Parallel Lines

The diagram shows triangle ABC.
CD is parallel to AB.
A, C and E lie in a straight line.
Angles of size a°, b° and c° are shown.



Not to scale

Insert a°, b° or c° to make this statement true. Give a reason for your answer.

Angle DCE = ...a°...... because .... alternate angles in parallel lines are equal.

Angles in Polygons

a) An interior angle of a regular polygon is eleven times its exterior angle.

Work out the number of sides of the polygon.

$12x = 180$

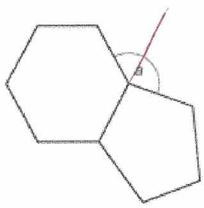
$x = \frac{180}{12} = 15°$

exterior angle

$\frac{360}{15} = 24$

.......24 sides.......

b) Imran joins two tiles together as shown below.
One tile is a regular hexagon and the other tile is a regular pentagon.



Not to scale

Imran thinks that another tile in the shape of a regular polygon will fit exactly into angle a.
Is Imran correct?
Show your reasoning.

$a = \frac{360}{6} + \frac{360}{5} = 60 + 72 = 132°$

supposed interior angle

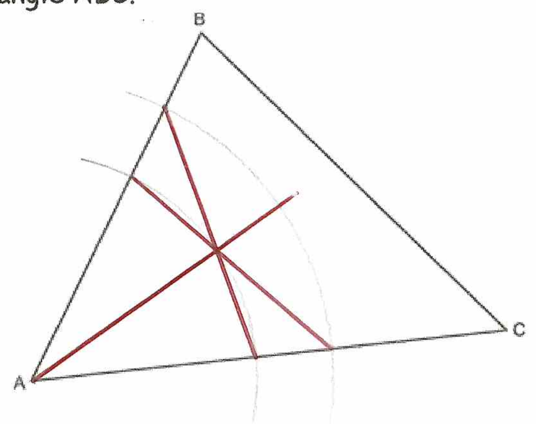
$180 - 132 = 48°$

$\frac{360}{48} = 7.5$

No shape can have 7.5 sides

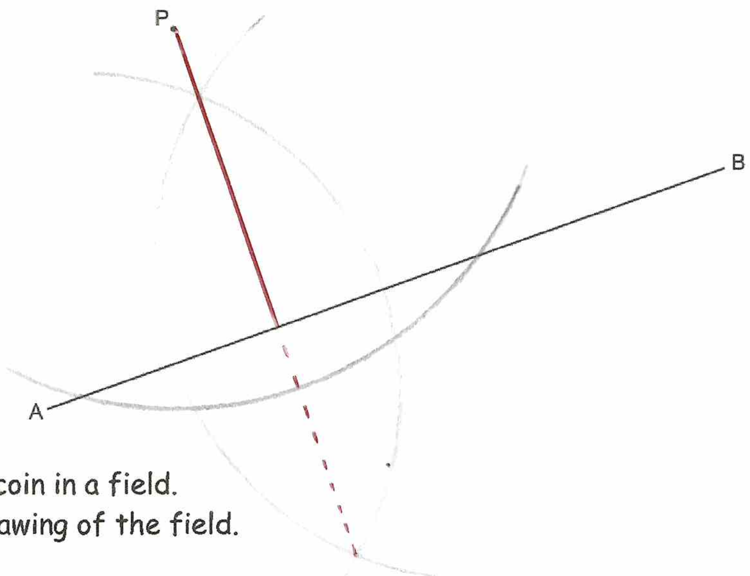
Constructions and Loci

a) The diagram shows triangle ABC.



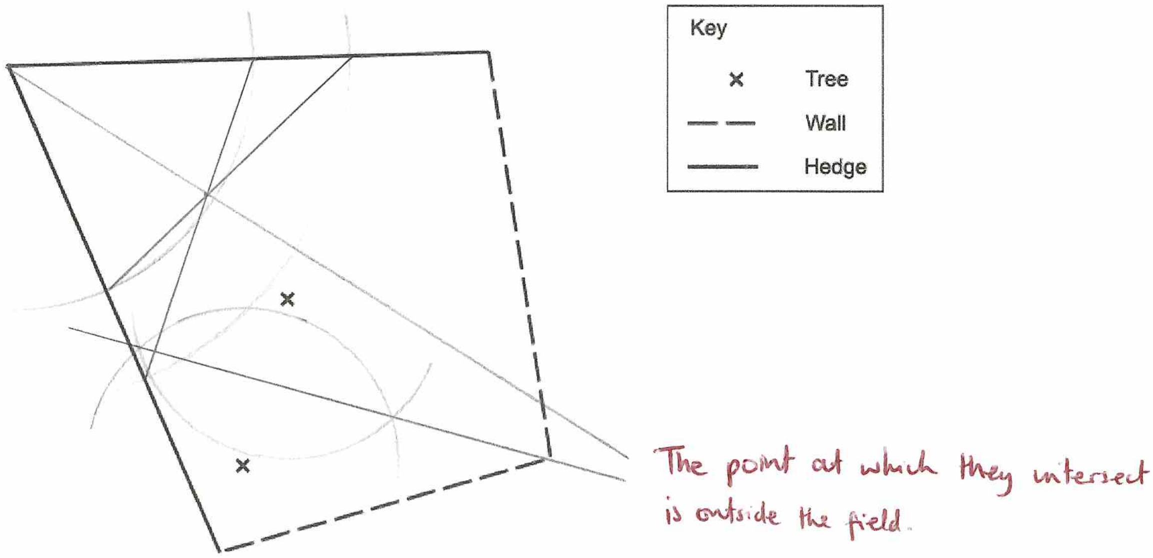
Construct the bisector of angle BAC.

b) Construct the perpendicular from the point P to the line AB.
Show all of your construction lines.



c) Jez finds a gold coin in a field.
This is a scale drawing of the field.

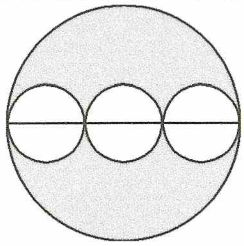
Scale: 1 cm represents 50 m



Jez says that the coin was an equal distance from each hedge and an equal distance from each tree.
Show by construction that Jez is wrong.

Area and Circumference of Circles

a) Three identical small circles are drawn inside one large circle, as shown in the diagram.
The centres of the small circles lie on the diameter of the large circle.



Find the fraction of the large circle that is shaded.

Assume $D = 6$, $d = 2$

$R = 3$, $r = 1$

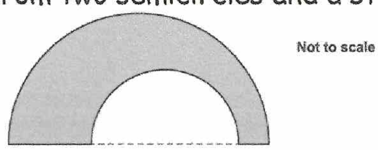
Area $= \pi R^2 = 9\pi$ $\quad \pi r^2 = \pi$

$\times 3 = 3\pi$

$9\pi - 3\pi = 6\pi$ $\quad \frac{6\pi}{9\pi} = \frac{2}{3}$

$\frac{2}{3}$

b) The shape below is formed from two semicircles and a straight line.



The radius of the large semicircle is 8 cm.
The radius of the small semicircle is t cm.

Find an expression, in terms of t, for the exact perimeter of the shaded shape.

Large $R = 8$ $D = 16$

Small $r = t$ $d = 2t$

Curve $= \frac{\pi D}{2} = \frac{16\pi}{2} = 8\pi$

Curve $= \frac{\pi d}{2} = \frac{2\pi t}{2} = \pi t$

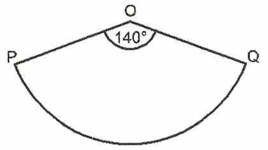
Straight $= D - d = 16 - 2t$

Perimeter $= 8\pi + \pi t + 16 - 2t$

$8\pi + \pi t + 16 - 2t$ cm

Sectors and Arc Lengths

a) OPQ is a sector of a circle, centre O and radius 9 cm.



Not to scale

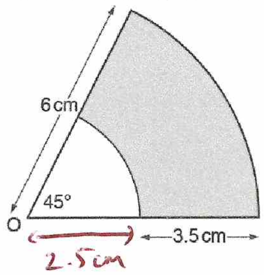
Find the perimeter of the sector.
Give your answer in terms of π.

Curve $= \frac{\theta}{360} \times \pi d = \frac{140}{360} \times \pi \times 18 = 7\pi$

Perimeter $=$ Curve $+$ radii $= 7\pi + 9 + 9$

$7\pi + 18$ cm

b) The design below is made from two sectors of circles, centre O.



Calculate the area of the shaded part.
Give your answer correct to 3 significant figures.

Area of a sector $= \frac{\theta}{360} \times \pi r^2$

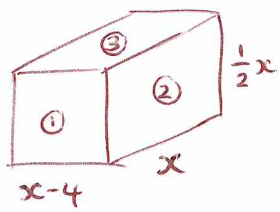
$\frac{45}{360} \times \pi \times 6^2 - \frac{45}{360} \times \pi \times 2.5^2$

$= 11.68279768$

11.7 cm^2

Surface Area

A cuboid has length x cm.
The width of the cuboid is 4 cm less than its length.
The height of the cuboid is half of its length.



Write an expression for the total surface area of the cuboid.

① $\frac{1}{2}x(x-4) = \frac{1}{2}x^2 - 2x$

② $\frac{1}{2}x \times x = \frac{1}{2}x^2$

③ $x(x-4) = x^2 - 4x$

$2x^2 - 6x$ $(\times 2)$

$4x^2 - 12x$ cm^2

Volume

a) A circular table top has radius 70cm.
The volume of the table top is 17,150π cm^3.
Calculate the thickness of the table top.

$V = \pi r^2 h$ $\pi \times 70^2 \times h = 17150\pi$

$4900h = 17150$

$h = \frac{17150}{4900} = 3.5$

......3.5cm......

a) A cuboid has length x cm.
The width of the cuboid is 4 cm less than its length.
The height of the cuboid is half of its length.
Work out the volume of the cuboid.

$x \times (x-4) \times \frac{1}{2}x$

$\frac{1}{2}x\,(x^2 - 4x) = \frac{1}{2}x^3 - 2x^2$

...$\frac{1}{2}x^3 - 2x^2$...cm^3

Cones and Spheres (1)

a) Calculate the total surface area of a cone with radius 5cm and slant height 12cm.

Curved surface area $= \pi r l = \pi \times 5 \times 12 = 60\pi$

$\pi r^2 = \pi \times 5^2 = 25\pi$

85π

......267.04......cm^2

b) A solid metal sphere has radius 9.8 cm.
The metal has a density of 5.023 g/cm^3.

Lynne estimates the mass of this sphere to be 20 kg.
Show that this is a reasonable estimate for the mass of the sphere.

$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 9.8^3 = 3942.45583$

Mass = Volume × Density

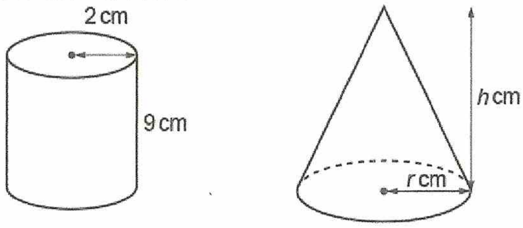
$= 3942.45583 \times 5.023 = 19802.95564g$

$\approx 19.8kg \approx 20kg$

....

Cones and Spheres (2)

c) The diagram shows a cylinder and a cone.



The cylinder has radius 2 cm and height 9 cm.
The cone has radius r cm and height h cm.

The ratio r : h is 1 : 4. $h = 4r$
The volume of the cone is equal to the volume of the cylinder.

Work out the value of r.

$\pi r^2 h = \pi \times 2^2 \times 9 = 36\pi$

$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times r^2 \times 4r = \frac{4}{3}\pi r^3$

$36\pi = \frac{4}{3}\pi r^3$

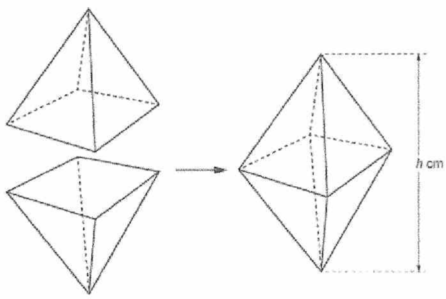
$108\pi = 4\pi r^3$

$108 = 4r^3$

$27 = r^3 \qquad r = \sqrt[3]{27}$

..........3..........

d) An octahedron is formed from two identical square based pyramids.
The square bases are stuck together as shown.



The volume of the octahedron is 60 cm^3.
The length of the side of each pyramid's square base is 5 cm.

Work out the height h cm of the octahedron.

$\frac{1}{3} \times$ area of base $\times$ height = ~~vola~~ volume of a pyramid

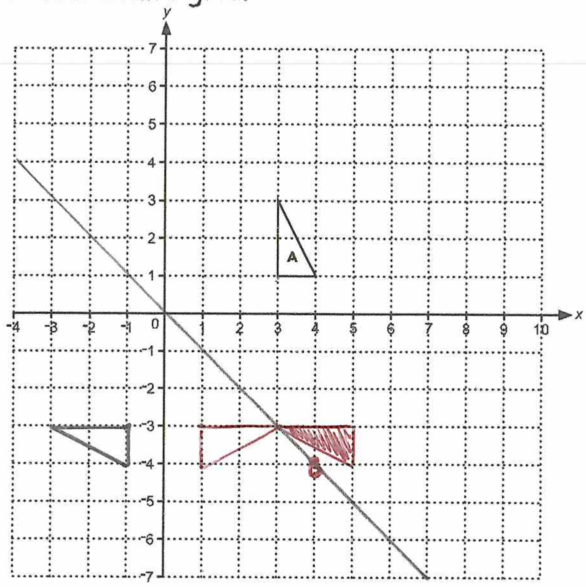
$\frac{1}{3} \times 25 \times \frac{1}{2}h = 30$

$\frac{25h}{6} = 30 \qquad 25h = 180 \qquad h = \frac{180}{25} = 7.2$

..........7.2cm..........

Transformations

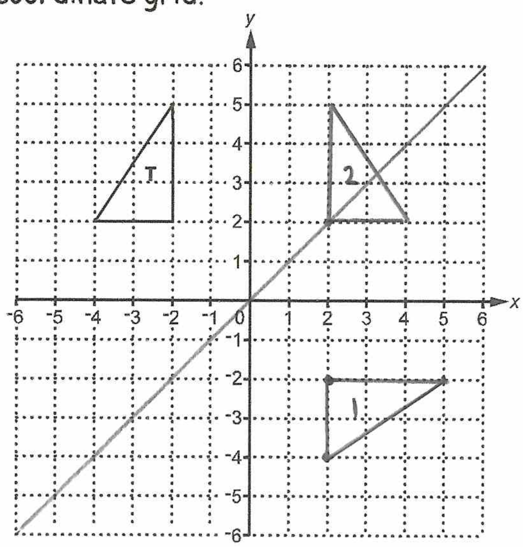
a) Triangle A is drawn on the coordinate grid.



Zara and Sam each transform triangle A onto triangle B.
Zara uses a rotation of 90° clockwise about the origin followed by a reflection in x = 3.
Sam uses a reflection in y = −x followed by a transformation T.
Describe fully transformation T.

Translation by the vector $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

b) A triangle T is drawn on a coordinate grid.

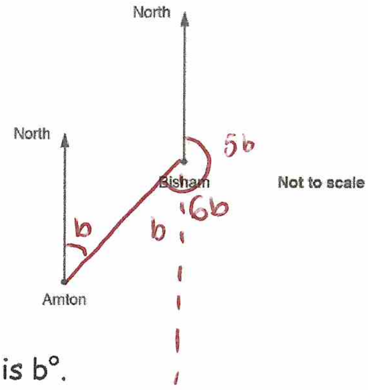


Describe fully the single transformation that is equivalent to a reflection in the line y = x, followed by a rotation of 90° anti-clockwise about (0, 0).

Reflection in the y-axis.

Bearings

The diagram shows the positions of two towns, Amton and Bisham.



The bearing of Bisham from Amton is b°.
The bearing of Amton from Bisham is 6b°.

Calculate the 3-figure bearing of Amton from Bisham.

$5b = 180$

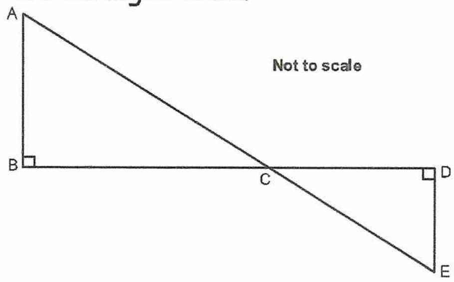
$b = \frac{180}{5} = 36$

$6b = 36 \times 6 = 216.$

..........216..........°

Similarity

In the diagram below, AE and BD are straight lines.



Show that triangles ABC and EDC are similar.

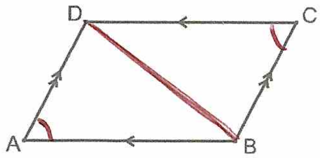
A | $C\hat{B}A = C\hat{D}E = 90°$

A | $A\hat{C}B = E\hat{C}D$ Vertically opposite angles are equal.

A | $B\hat{A}C = C\hat{E}D$ Alternate angles in parallel lines are equal.

Congruent Triangles

ABCD is a parallelogram.



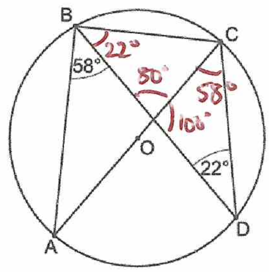
Prove that triangle ABD is congruent to triangle CDB.

S | $AD = BC$ Equal lengths in a parallelogram
A | $B\hat{A}D = D\hat{C}B$ Opposite angles in a parallelogram are equal.
S | $AB = CD$ Equal lengths in a parallelogram.

Congruent by SAS.

Circle Theorems (1)

a) A, B, C and D are points on the circumference of a circle, centre O.
AC is a diameter of the circle.
Angle ABD = 58°.
Angle CDB = 22°.



Not to scale

Find the size of angle ACD, giving reasons for your answers.

58°. Angles in the same segment from a common chord to the circumference are equal.

Find the size of angle ACB, giving reasons for your answers.

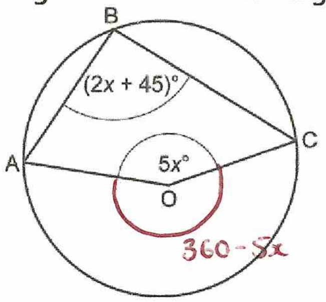
$C\hat{B}A = 90°$. Angles subtended by a diameter are right angles at the circumference.

$D\hat{B}C = 90 - 58 = 22°$.

$180 - (80 + 22) = 78°$.

b) A, B and C lie on a circle, centre O.

In quadrilateral ABCO, angle AOC = $5x°$ and angle ABC = $(2x + 45)°$.



Find the value of x.

$2(2x+45) = 360 - 5x$

$4x + 90 = 360 - 5x$

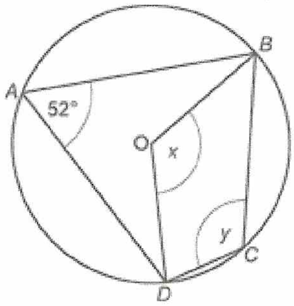
$9x + 90 = 360$

$9x = 270$

$x = 30$

$x = 30.$

c) A, B, C and D are points on the circumference of a circle, centre O.



Angle BAD = 52°.

Work out the size of angles x and y. Give reasons for your answers.

$y = 180 - 52 = 128°$ Opposite angles in a cyclic quadrilateral add up to 180°

$x = 52° \times 2 = 104°$ Angles at the centre are double the angle at the circumference.

Vectors

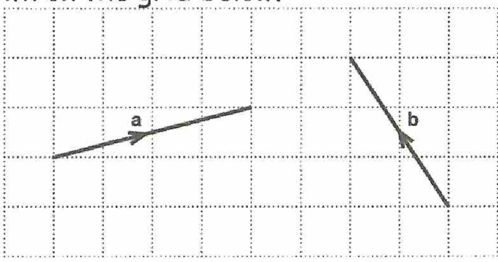
a) Work out

$$\begin{pmatrix}3\\4\end{pmatrix} - 2\begin{pmatrix}1\\-3\end{pmatrix}$$

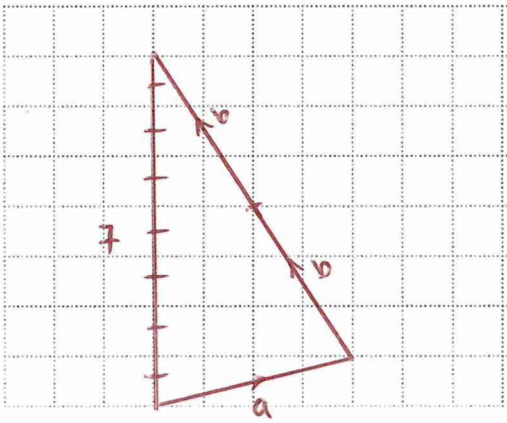
$$\begin{pmatrix}3\\4\end{pmatrix} - \begin{pmatrix}2\\-6\end{pmatrix} = \begin{pmatrix}1\\10\end{pmatrix}$$

.... $\begin{pmatrix}1\\10\end{pmatrix}$ ....

b) Two vectors, a and b, are shown on the grid below.



Show that the vector a + 2b has a length of 7 units.
You may use the grid below.

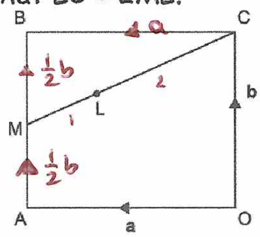


$a = \begin{pmatrix}4\\1\end{pmatrix}$ $\quad b = \begin{pmatrix}-2\\3\end{pmatrix}$

$$\begin{pmatrix}4\\1\end{pmatrix} + 2\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix} + \begin{pmatrix}-4\\6\end{pmatrix} = \begin{pmatrix}0\\7\end{pmatrix}$$

c) OABC is a square.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.
M is the midpoint of AB.
L is a point on MC such that LC = 2ML.



Not to scale

Use vectors to prove that the point L lies on the line OB.

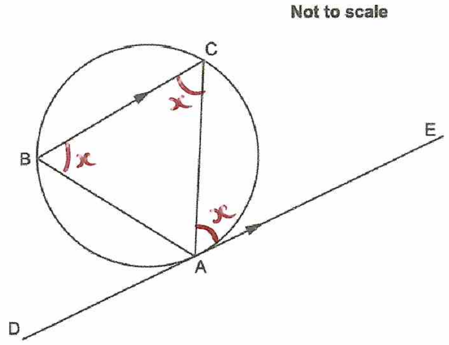
$$\begin{aligned}\overrightarrow{OL} &= a + \tfrac{1}{2}b + \tfrac{1}{3}\overrightarrow{ML}\\ &= a + \tfrac{1}{2}b + \tfrac{1}{3}\left(\tfrac{1}{2}b - a\right)\\ &= a + \tfrac{1}{2}b + \tfrac{1}{6}b - \tfrac{1}{3}a\\ &= \tfrac{2}{3}a + \tfrac{2}{3}b\end{aligned}$$

$$\begin{aligned}\overrightarrow{LB} &= \tfrac{2}{3}\overrightarrow{ML} + a\\ &= \tfrac{2}{3}\left(\tfrac{1}{2}b - a\right) + a\\ &= \tfrac{1}{3}b - \tfrac{2}{3}a + a\\ &= \tfrac{1}{3}a + \tfrac{1}{3}b\end{aligned}$$

OLB is a straight line, as $\overrightarrow{OL}$ and $\overrightarrow{LB}$ are parallel and share a common point.

Geometric Proof

a) The diagram shows points A, B and C on the circumference of a circle.
Line DAE is a tangent to the circle.
DE is parallel to BC.



Prove that triangle ABC is an isosceles triangle.
Give the reason for each step in your proof.

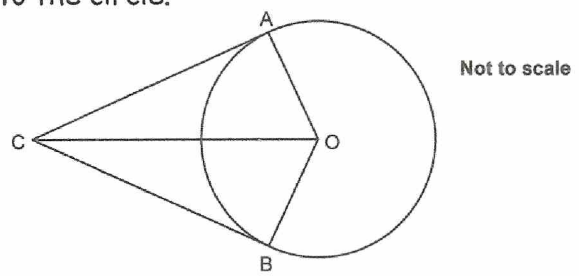
Let $E\hat{C}A = x^\circ$

$B\hat{C}A = x^\circ$, ~~abt~~ alternate angles in parallel lines are equal.

$A\hat{B}C = x^\circ$, alternate segment theorem.

$B\hat{C}A = A\hat{B}C\ (= x^\circ)$, so ΔABC is isosceles.

a) A and B are points on the circumference of a circle, centre O.
CA and CB are tangents to the circle.



Prove that triangle OAC is congruent to triangle OBC.

$AC = BC$, tangents from the same point to the same ~~length~~ circle are equal in length

$OA = OB$, radii in the same circle

$C\hat{A}O = O\hat{B}C = 90^\circ$, radii meet tangents at right angles

Congruent via
S | $AC = BC$
A | $C\hat{A}O = O\hat{B}C$
S | $OA = OB$

END OF QUESTIONS