

# RATIO AND PROPORTION

## Sharing in a Ratio

- a) Peter makes a large amount of pink paint by mixing red and white paint in the ratio 2 : 3.  
Red paint costs £80 per 10 litres.  
White paint costs £5 per 10 litres.

Peter sells his pink paint in 10-litre tins for £60 per tin.  
Calculate how much profit he makes for each tin he sells.

$$\text{Red} = \text{£}8 \text{ per litre}$$

$$\text{White} = \text{£}0.50 \text{ per litre}$$

$$10\text{-litres of pink paint} = 4\text{L red and } 6\text{L white}$$

$$4 \times \text{£}8 + 6 \times \text{£}0.50 = \text{£}35$$

$$\begin{array}{r} \text{£}60 \\ - \text{£}35 \\ \hline \text{£}25 \end{array}$$

.....  
£25

- b) A bakery bakes small, medium and large pies.  
The ratio small : medium : large is 3 : 5 : 2.  
One day 460 medium pies are baked.  
Work out how many small pies are baked.

$$5 \text{ parts} = 460$$

$$1 \text{ part} = 460 \div 5 = 92$$

$$\text{Small} = 3 \text{ parts} = 3 \times 92 = 276$$

.....  
276

## Three Part Ratios

Anne, Barry and Colin share a prize in the ratio 3 : 4 : 5.  
Colin gives  $\frac{1}{3}$  of his share to a charity.

What fraction of the whole prize does Colin give to the charity?

$$A : B : C = 3 : 4 : 5 = 9 : 12 : 15$$

↓

$$5 \text{ parts to charity out of } 9 + 12 + 15 \\ = 36$$

.....  
 $\frac{5}{36}$

### Writing Ratios as Fractions

In a school,  $\frac{2}{3}$  of the students study a language.

Of those students who study a language,  $\frac{2}{5}$  study Spanish.

Find the ratio of students who study Spanish to students who do not study Spanish.

Study : Don't Study  
2 : 3  
= 10 : 15  
↓  
 $\frac{2}{5}$  study Spanish = 4 parts. 6 parts don't + 15 don't study a language 4:21

### Exchange Rates

Tony returns from holiday with two €50 notes, four €20 notes, 9 €10 notes and 12 €5 notes.

The exchange rate is £1 = €1.17.

Work out how much he will get in total when he changes these notes.

$$2 \times \text{€}50 + 4 \times \text{€}20 + 9 \times \text{€}10 + 12 \times \text{€}5 = 100 + 80 + 90 + 60 = \text{€}330$$

$$\frac{330}{1.17} = 282.051282051$$

£282.05

### Best Value

Tea bags are sold in three different sized packs:

Small Pack	Medium pack	Large pack
80 tea bags for £2.10	150 tea bags for £3.55	220 tea bags for £5.25

Which size pack offers the best value for money?

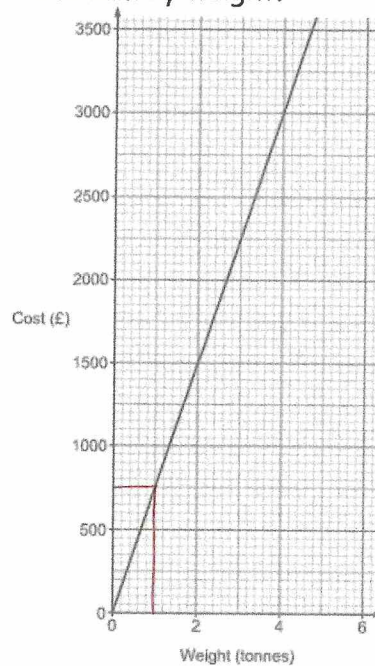
1 tea bag =  $\frac{\text{£}2.10}{80}$       $\frac{\text{£}3.55}{150}$       $\frac{\text{£}5.25}{220}$

~~2.625p~~     2.36p     2.3863p

Medium pack

## Conversion Graphs

The graph below shows the cost of aluminium by weight.



1 tonne = £750

Work out the cost of 17 tonnes of aluminium.

$$£750 \times 17$$

£12750

What assumption have you made about the cost of aluminium in your calculations?

That each tonne costs the same, regardless of the amount of aluminium bought. That is, no discounts for bulk buying.

## Simple and Compound Interest

Here are the interest rates for two accounts.

Account A	Account B
Interest: 3% per year compound interest.	Interest: 4% for the first year, 3% for the second year and 2% for the third year.
No withdrawals until the end of three years.	Withdrawals allowed at any time.

Derrick has £10,000 he wants to invest.

Calculate which account would give him most money if he invests his money for 3 years.

Give the difference in the interest to the nearest penny.

$$A: £10000 \times 1.03^3$$

$$= £10927.27$$

$$B: £10000 \times 1.04 \times 1.03 \times 1.02$$

$$= £10926.24$$

£1.03 difference.

## Depreciation

a) Amelia buys a new car.

The expected future value of this car, £V, is given by

$$V = 36000 \times 0.72^t$$

where t is the age of the car in complete years.

Write down the value of the car when new.

£ 36,000 .....

Write down the annual percentage decrease in the expected value of the car.

..... 28 %

Calculate the expected value of the car after 4 years.

$$36000 \times 0.72^4 = 9674.58816$$

£ 9674.59 .....

b) In 2017, the value of a house increased by 4%.

In 2018, the value of the house decreased by 3%.

Teresa says,

"Over the two years the value of the house increased by exactly 1% because  $4 - 3 = 1$ ."

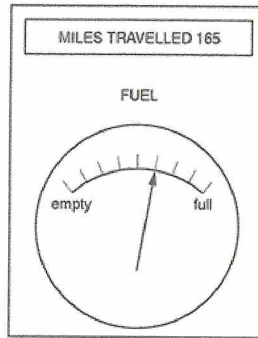
Show that Teresa is wrong.

$$x \times 1.04 \times 0.97 = x \times 1.0088$$

↑  
Multiplier increases by 0.88%, not 1%.

## Direct Proportion

- a) Ifsaw noticed this information on her car's dashboard at the end of her journey.  
She started her journey with a full tank of fuel and her miles travelled set to zero.



Work out how far Ifsaw's car can travel on a full tank of fuel.

$\frac{3}{8}$  used to travel 165 miles.

$\frac{1}{8}$  would travel 55 miles

$$55 \times 8 = 440$$

440 miles

- b)  $y$  is directly proportional to  $\sqrt{x}$ .  
 $y$  is 75 when  $x = 100$ .

Find a formula linking  $x$  and  $y$ .

$$y = k\sqrt{x}$$

$$75 = k \times \sqrt{100}$$

$$\frac{75}{\sqrt{100}} = k$$

$$k = 7.5$$

$y = 7.5\sqrt{x}$



## Inverse Proportion

a)  $y$  is inversely proportional to the square of  $x$ .

Complete the table:

$x$	10	6	15
$y$	9	25	4

$$y = \frac{k}{x^2}$$

$$y = \frac{900}{6^2}$$

$$4 = \frac{900}{x^2}$$

$$9 = \frac{k}{10^2}$$

$$y = \frac{900}{36}$$

$$x^2 = \frac{900}{4} = 225$$

$$k = 900$$

$$y = \frac{900}{x^2}$$

$$x = \sqrt{225} = 15$$

b) At a constant temperature, the volume of a gas  $V$  is inversely proportional to its pressure  $p$ .  
By what percentage will the pressure of a gas change if its volume increases by 25%?

$$V \propto \frac{1}{p} \quad V = \frac{k}{p}$$

$$V \times p = k$$

$$V \times 1.25 \times p \times ? = k$$

$$? = \frac{1}{1.25} = 0.8$$

20% decrease

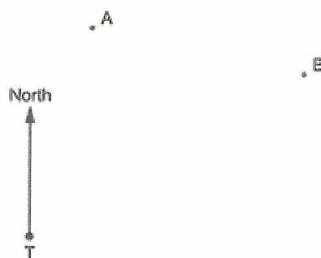
## Speed, Distance, Time

T is a radar tower.

A and B are two aircraft.

At 3pm, aircraft A is 3250 km from T on a bearing of  $015^\circ$  and aircraft B is 4960 km from T on a bearing of  $057^\circ$ .

Not to scale



Aircraft A flies directly towards radar tower T at a speed of 890 km/h.

At what time will the aircraft pass over radar tower T?

Give your answer to the nearest minute.

$$S = \frac{d}{t} \quad S \times t = d \quad t = \frac{d}{S}$$

$$\text{Time} = \frac{3250 \text{ km}}{890 \text{ km/h}} = 3.651685393$$

$$= 3 \text{ hrs and } 0.651685393 \times 60 \text{ minutes}$$

$$= 3 \text{ hrs and } 39.10123596 \text{ minutes}$$

6:39pm

## Density, Mass, Volume

180 g of copper is mixed with 105 g of zinc to make an alloy.

The density of copper is  $9 \text{ g/cm}^3$ .

The density of zinc is  $7 \text{ g/cm}^3$ .

What is the density of the alloy?

	D	M	V
Copper	$9 \text{ g/cm}^3$	180g	$20 \text{ cm}^3$
Zinc	$7 \text{ g/cm}^3$	<u>105g</u>	<u><math>15 \text{ cm}^3</math></u>
		285g	$35 \text{ cm}^3$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{285\text{g}}{35\text{cm}^3} = 8.142857143$$

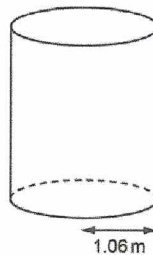
8.14.....  $\text{g/cm}^3$

## Pressure, Force, Area

Weight is measured in newtons (N).

A cylinder of ice of weight 5940 N rests on a horizontal surface.

The base of the cylinder has radius 1.06 m.



Hannah estimates that the pressure exerted by the cylinder on the surface is  $1000 \text{ N/m}^2$ .

Show that Hannah's estimate is incorrect.

$$r \approx 1 \quad \pi r^2 \approx \pi \approx 3$$

$$\text{Pressure} = \frac{\text{force}}{\text{Area}} \approx \frac{6000 \text{ N}}{3 \text{ m}^2} = 2000 \text{ N/m}^2$$

Hannah's estimate is incorrect

.....

## Velocity-Time Graphs

A toy car is placed on the floor of a sports hall.

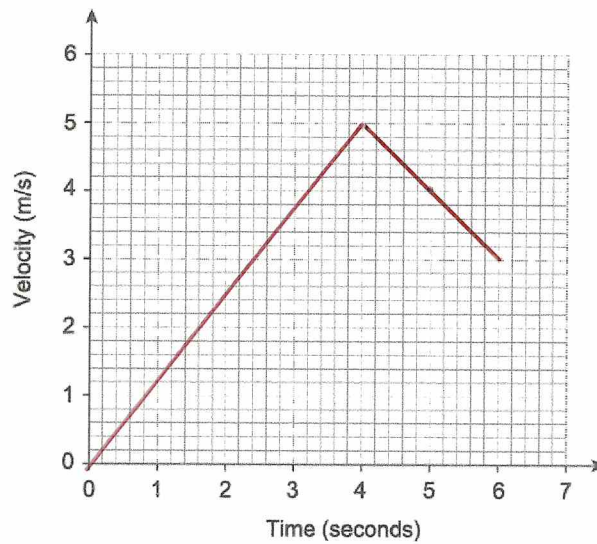
It moves in a straight line starting from rest.

It travels with constant acceleration for 4 seconds reaching a velocity of 5 m/s.

It then slows down with constant deceleration of  $1 \text{ m/s}^2$  for 2 seconds.

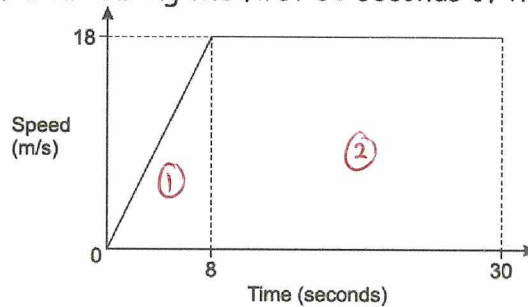
It then hits a wall and stops.

Draw a velocity-time graph for the toy car.



## Area Under a Graph

The graph shows the speed of a car during the first 30 seconds of its journey.



Find the total distance travelled by the car in the 30 seconds.

$$\textcircled{1} = \frac{8 \times 18}{2} = 72 \text{ m}$$

$$\textcircled{2} = 22 \times 18 = 396 \text{ m}$$

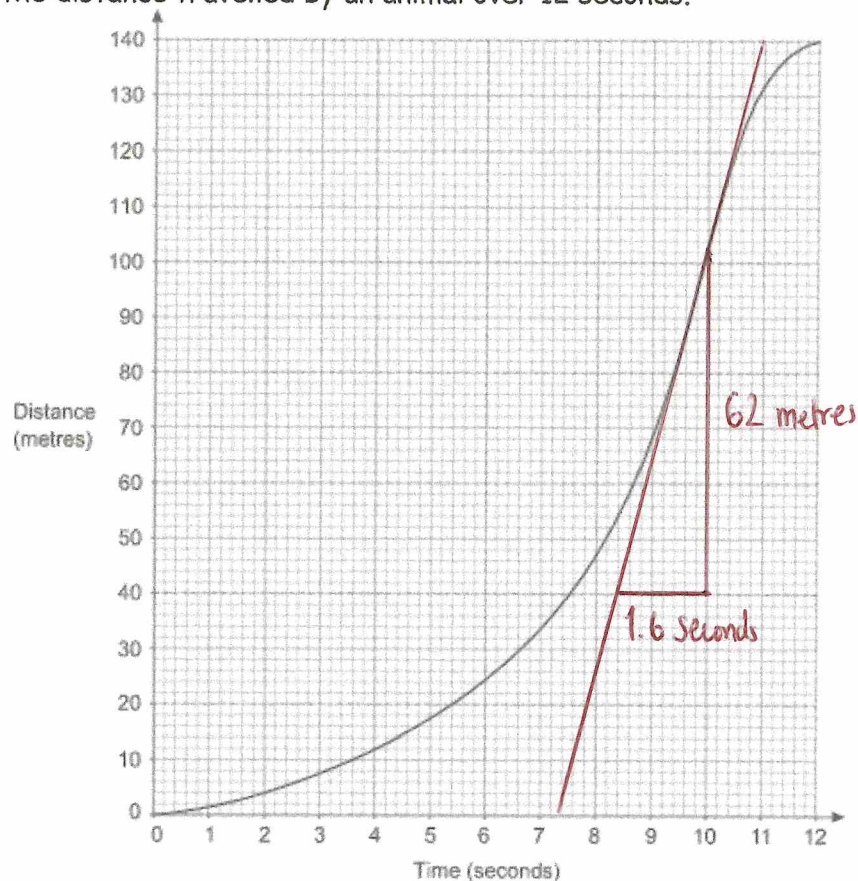
$$72 + 396 = 468 \text{ m}$$

468 m



## Gradient of a Graph

The graph shows the distance travelled by an animal over 12 seconds.



Nuri says,

"I think this animal must be able to move at over 20 m/s!"

Do you agree with Nuri?

Explain your reasoning.

Speed is estimated by drawing a tangent, and finding its gradient

Estimated speed at 10s is  $\frac{62}{1.6} = 38.75 \text{ m/s}$ .

I agree with Nuri.

## Equating Ratios

- a) A bag of sweets contains only mints, sherberts and toffees.

The ratio of the number of mints to sherberts is 2 : 3.

The ratio of the number of sherberts to toffees is 7 : 5.

What fraction of the sweets are sherberts?

$$\begin{array}{l} M:S \\ 2:3 \\ S:T \\ 7:5 \\ \hline 14:21 \\ 21:15 \\ \hline 14:21:15 \end{array}$$

$$\frac{21}{50}$$

- b) There is a total of 250 men, women and children on a train.

The ratio of men to women is 4 : 5.

The ratio of women to children is 10 : 7.

How many men are on the train?

$$\begin{array}{l} M:W \\ 4:5 \\ W:C \\ 10:7 \\ \hline 8:10 \\ 10:7 \end{array}$$

$$8:10:7 = 25 \text{ parts} \quad \frac{250}{25} = 10$$

$$80:100:70$$

$$\frac{80}{250}$$